9 Areas and Perimeters

This is is our next key Geometry unit. In it we will recap some of the concepts we have met before. We will also begin to develop a more algebraic approach to finding areas and perimeters.
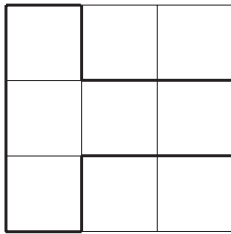
9.1 Area

The easiest method to find an area of a shape, particularly if it is a simple shape made up of straight lines, is to count the squares inside it.
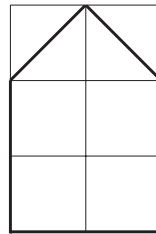
Example 1

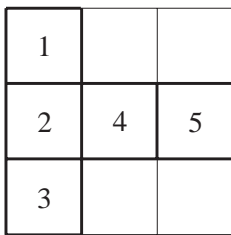Find the area of each of these shapes in terms of the square shown.
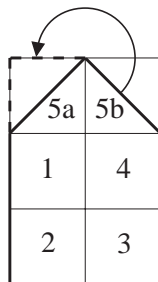
(a)

(b)

One square

Solution

(a)

This can be divided into 5 of the squares, so its area is 5 square units.
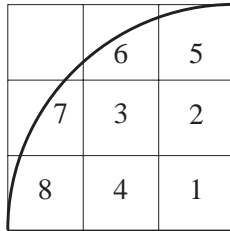
(b)

We have 4 squares (labelled 1, 2, 3 and 4), and the two triangles (labelled 5a and 5b) can be joined together to form another square. So, in total, we have an area of 5 square units.

Example 2

Estimate the area of this shape.

Solution

There are 4 complete squares (labelled 1, 2, 3 and 4). Region 5 and 6 together make up about $1\frac{1}{2}$ squares, as do regions 7 and 8. So we have another 3 squares giving a total of 7 square units (plus a little bit more!).
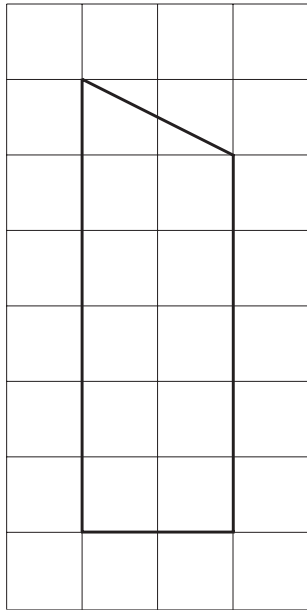
Exercises

1. Draw around your hand on squared paper and find its area.

    Who has the largest hand in your class?

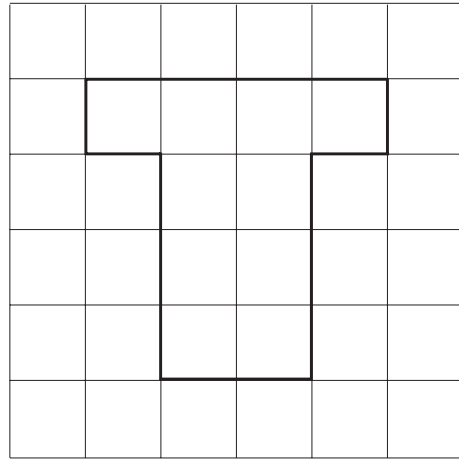2. Find the area of this circle by counting squares.
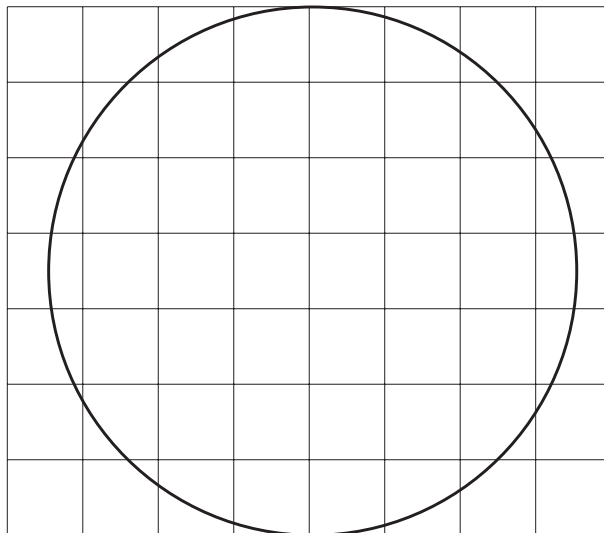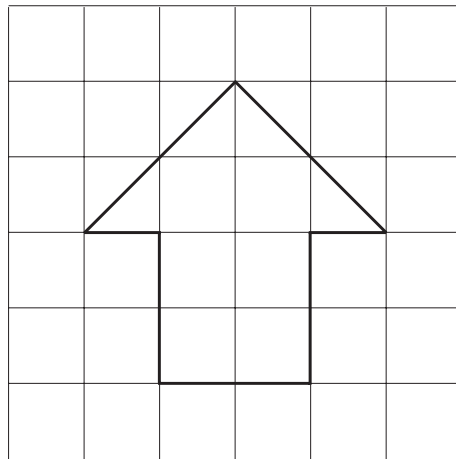
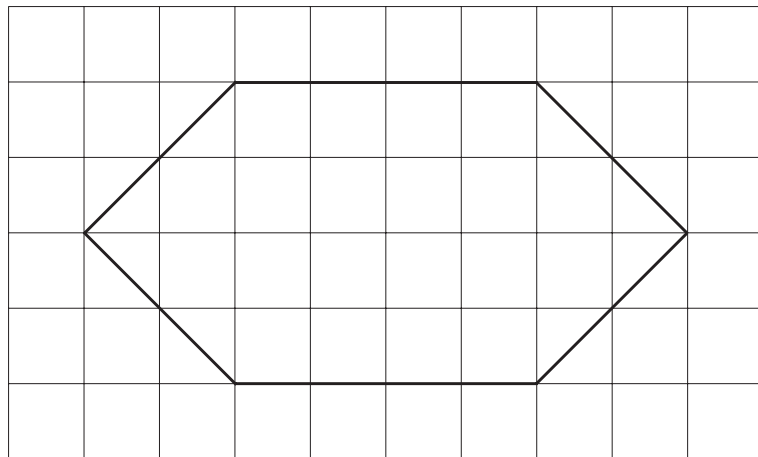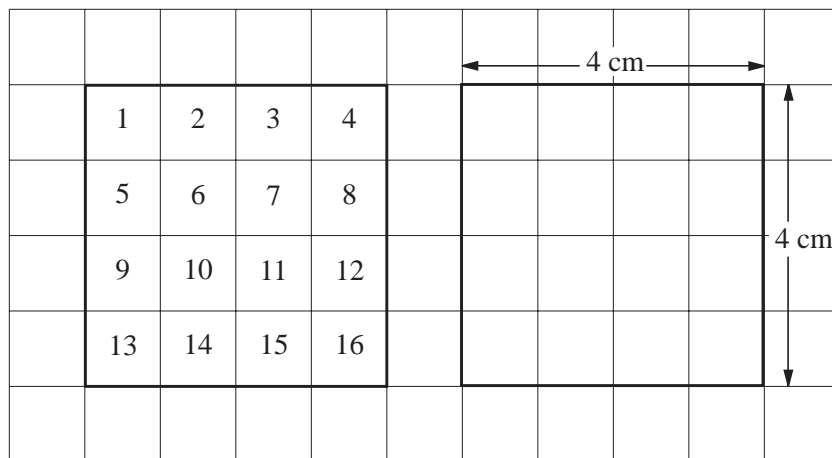3.　Find the areas of these shapes by counting squares.

(a)

(b)

(c)

(d)

9.2 Area and Perimeter of a Square

We now bring in standard units for measuring area and perimeter. You should always put units in your answers.

The area of a square can be found by counting squares or multiplying the length of the sides. The area of a square with sides 1 cm is 1 cm^2.
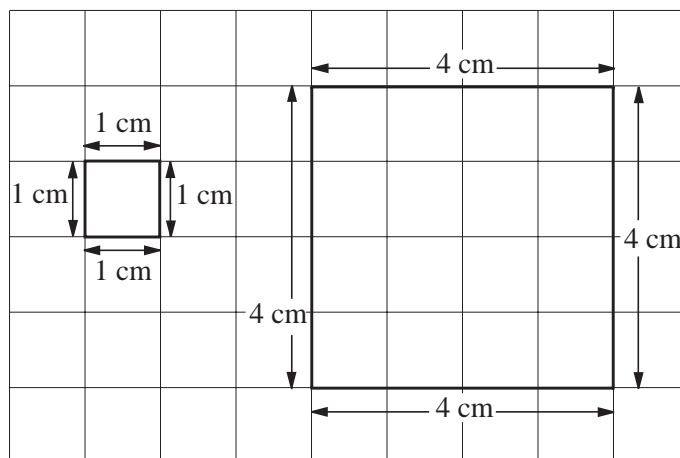
Area $= 1 \text{ cm}^2$

Area $= 16 \text{ cm}^2$

Area $= 4 \times 4$

$= 16 \text{ cm}^2$

The perimeter of a square is the total length of the four sides.

Perimeter $= 1 + 1 + 1 + 1$
$= 4 \text{ cm}$

Perimeter $= 4 + 4 + 4 + 4$
$= 16 \text{ cm}$

♪ ## Note

Note also that:

1 m	=	100 cm
1 cm	=	10 mm

So that, for example,

$$25 \text{ mm} = 2.5 \text{ cm}$$
$$8 \text{ mm} = 0.8 \text{ cm}$$
$$261 \text{ cm} = 2.61 \text{ m}$$
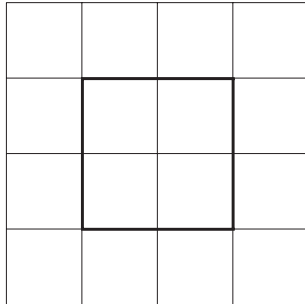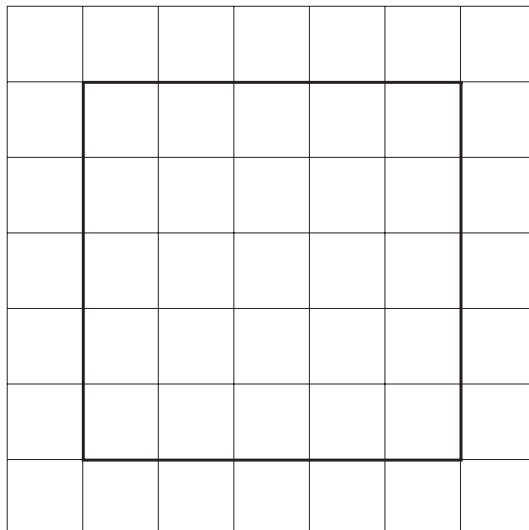$$32 \text{ cm} = 0.32 \text{ m}$$
$$6 \text{ cm} = 0.06 \text{ m}$$

Exercises

1. Find the area and perimeter of each of these squares.

   (a)
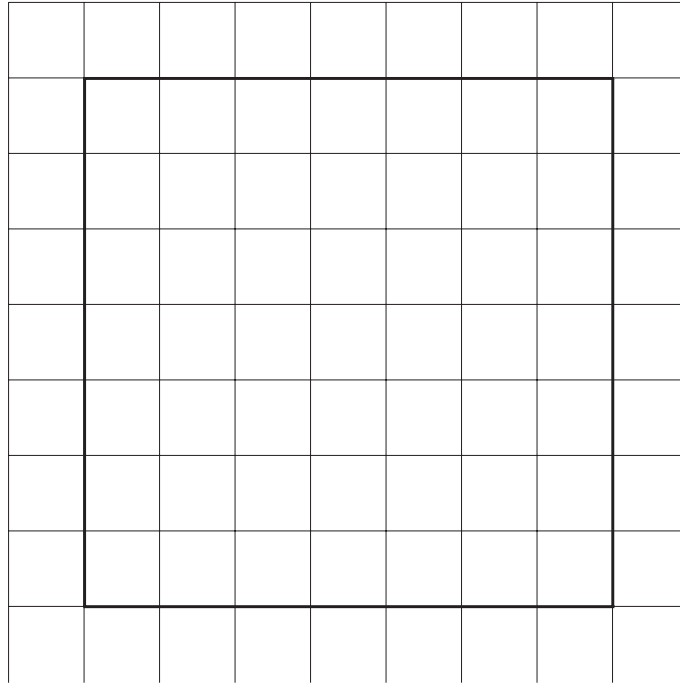
   (b)

(c)

2. Find the area of squares with sides of length:

   (a) 10 cm   (b) 12 cm   (c) 8 cm

   (d) 9 cm   (e) 15 cm   (f) 20 cm

3. Find the perimeter of squares with sides of length:

   (a) 13 cm   (b) 8 cm   (c) 16 cm

   (d) 19 cm   (e) 9 cm   (f) 18 cm

4. Copy and complete each of these statements.

   (a) 3.2 cm = ⬜ mm   (b) 10.3 cm = ⬜ mm

   (c) 28 mm = ⬜ cm   (d) 216 mm = ⬜ cm

   (e) 152 cm = ⬜ m   (f) 84 cm = ⬜ m

   (g) 1.62 m = ⬜ cm   (h) 1.7 m = ⬜ cm

   (i) 0.82 m = ⬜ cm   (j) 0.07 m = ⬜ cm

5. A square has sides of length 20 mm. Find the area of the square in:

   (a) mm^2 (b) cm^2

6. The perimeter of a square is 40 cm. How long are its sides?

7.   The area of a square is 36 cm^2. How long are its sides?

8.   The perimeter of a square is 44 cm. What is its area?

9.   The area of a square is 144 cm^2. What is its perimeter?

10.   For a 2 cm square the perimeter is 8 cm and the area is 4 cm^2. The perimeter is twice the area.

        What are the lengths of the sides of a square for which the perimeter is

        (a)   equal to the area;

        (b)   half of the area?

9.3 The Area and Perimeter of a Rectangle

For a rectangle, say 5 cm by 2 cm, we can proceed either by counting squares or multiplying the lengths. So for example,
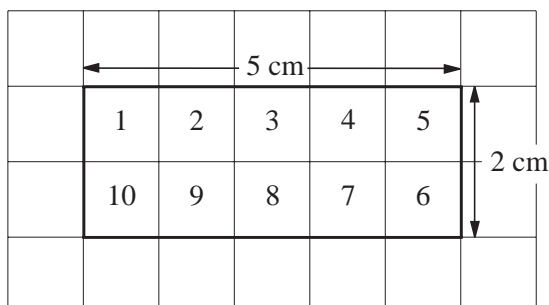
the area of this rectangle is 10 cm^2 from counting squares or, alternatively;

$$\text{Area} = 5 \times 2$$

$$= 10 \text{ cm}^2$$

Note also that 1 cm is the same as 10 mm,

so that a 1 cm square has an area of 1 cm^2 and this can also be written as

$$1 \text{ cm} \times 1 \text{ cm} = 10 \text{ mm} \times 10 \text{ mm}$$

i.e $\boxed{1 \text{ cm}^2 = 100 \text{ mm}^2}$

9.3

Example

What is 1 m^2 in terms of cm^2?

Solution

$$1 \text{ m} \times 1 \text{ m} = 100 \text{ cm} \times 100 \text{ cm}$$

i.e. $1 \text{ m}^2 = 10000 \text{ cm}^2$

Exercises

1. Find the area of these rectangles in cm^2.
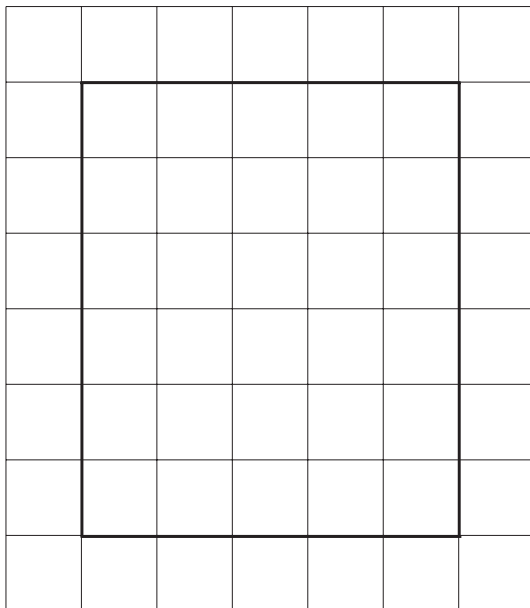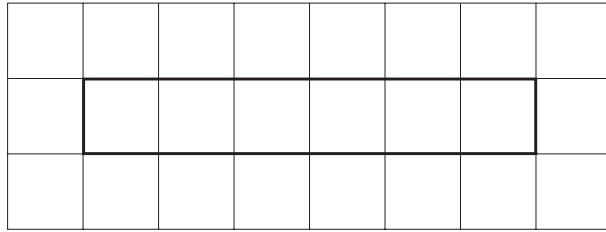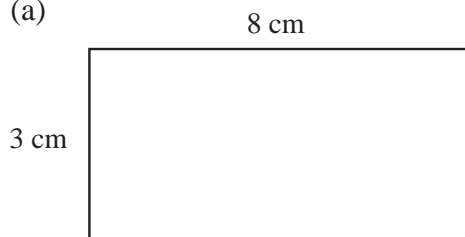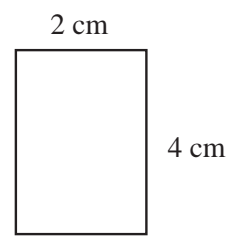
(a)
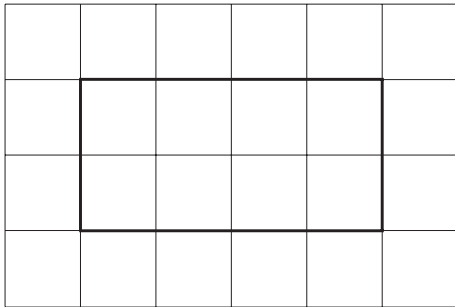
(b)
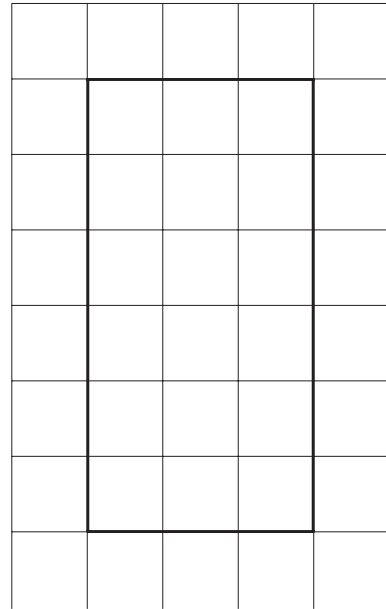
(c)

(d)

2. Find the perimeter of the rectangles in question 1.

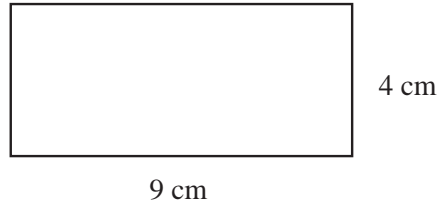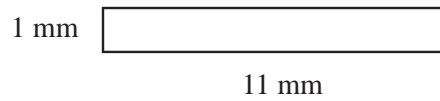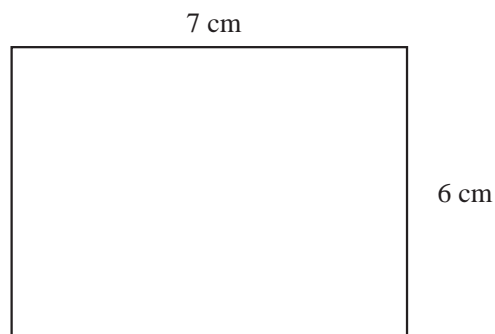3. Find the area of these rectangles in suitable units. The diagrams have not been drawn accurately.
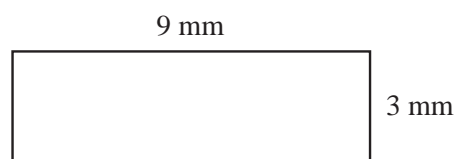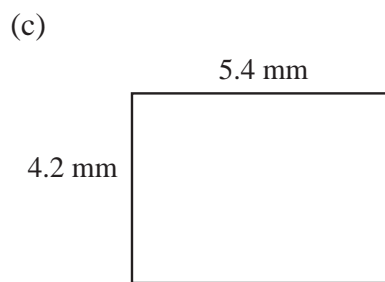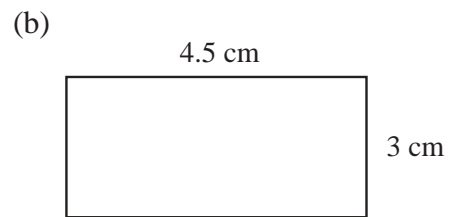
(a)

8 cm

3 cm

(b)

2 cm

4 cm

(c)

4 cm

9 cm

(d)

1 mm

11 mm

(e)

7 cm

6 cm

(f)

9 mm

3 mm

9.3

4. Find the perimeter of the rectangles in question 3.

5. Find the area and perimeter of these rectangles.

(a)

6.2 cm

4 cm

(b)

4.5 cm

3 cm

(c)

5.4 mm

4.2 mm

(d)

1.5 m

1.4 m

(e)

3.6 cm

6.1 cm

(f)

7.4 mm

8 mm
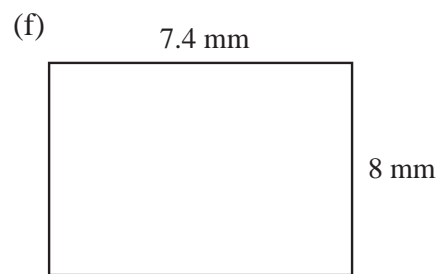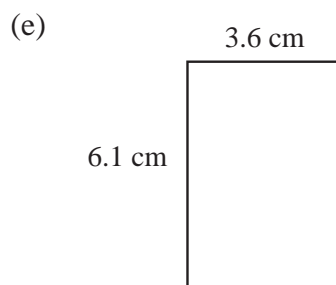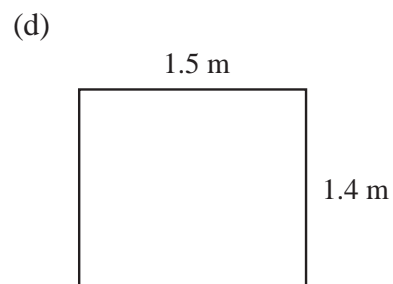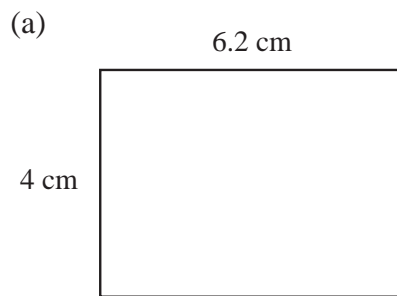
6. Find the area and perimeter of this rectangle

(a) in cm^2 and cm

(b) in m^2 and m.

1 m

30 cm
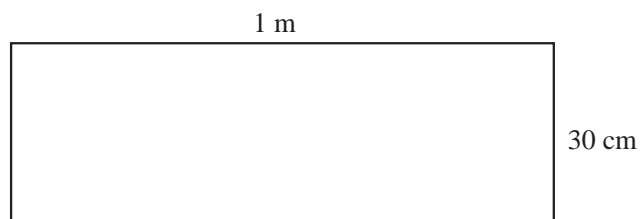
7. Find the area of this rectangle in mm^2 and cm^2.

5 cm

3 cm
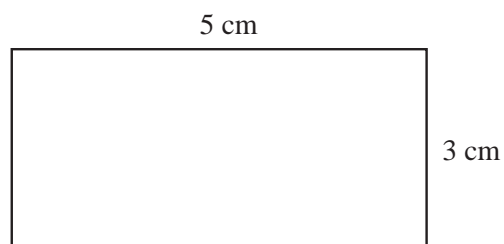
8. Find the perimeter and area of this rectangle making clear which units you have decided to use.

1.5 cm

8 mm
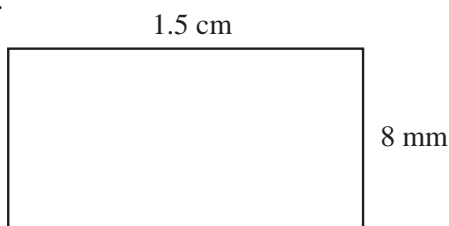
9. A rectangle has an area of 48 cm^2. The length of one side is 6 cm. Find the perimeter of the rectangle.

10. A rectangle has a perimeter of 24 cm and an area of 32 cm^2. What are the lengths of the sides of the rectangle?
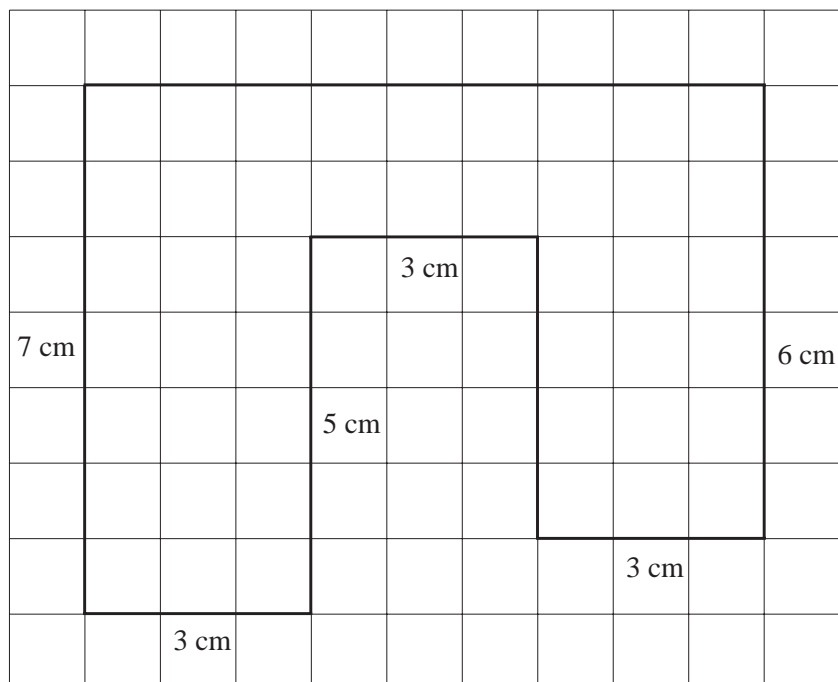
9.4 Area of Compound Shapes
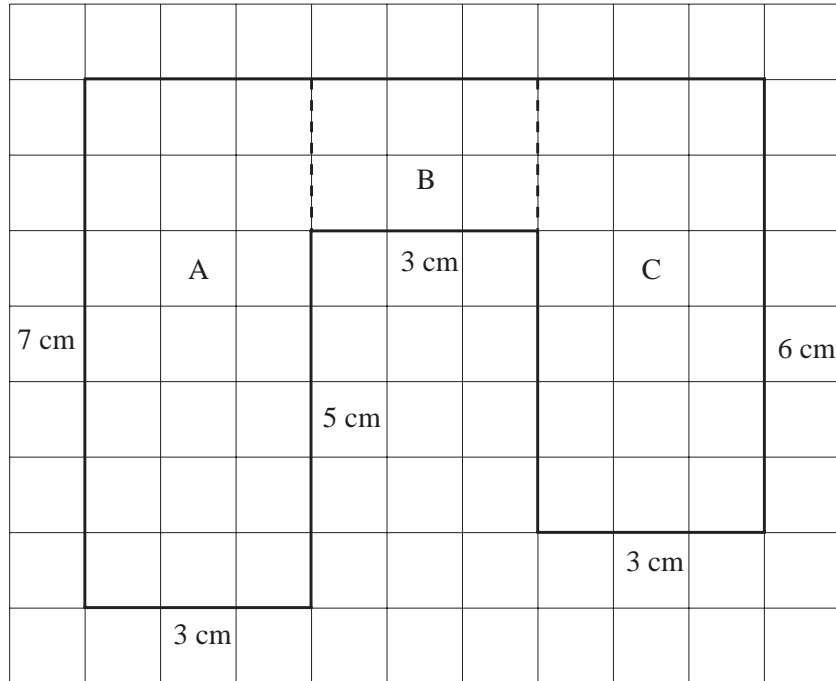
We illustrate this method with an example.

Example

Find the area of the shape shown below.

3 cm

7 cm

6 cm

5 cm

3 cm

3 cm

Solution

Divide the shape into rectangles; one way is shown below.

$$\text{Area of A} = 3 \times 7 = 21 \text{ cm}^2$$

$$\text{Area of B} = 3 \times 2 = 6 \text{ cm}^2$$

$$\text{Area of C} = 3 \times 6 = 18 \text{ cm}^2$$

$$\textit{Total Area} = 21 + 6 + 18 = 45 \text{ cm}^2$$
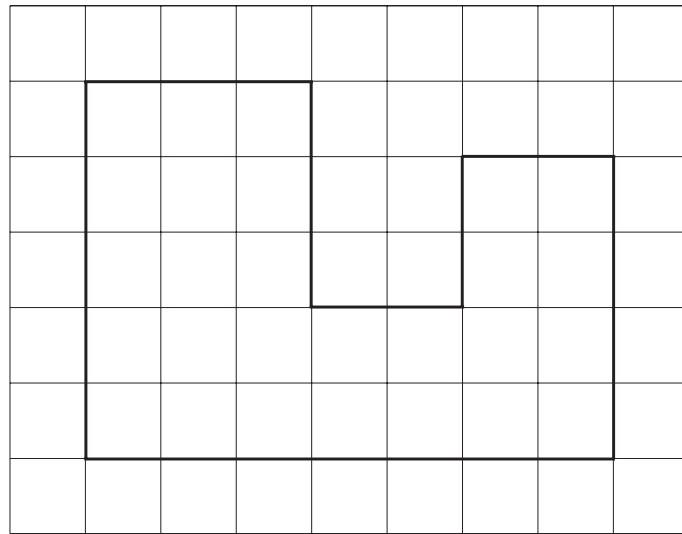
Exercises
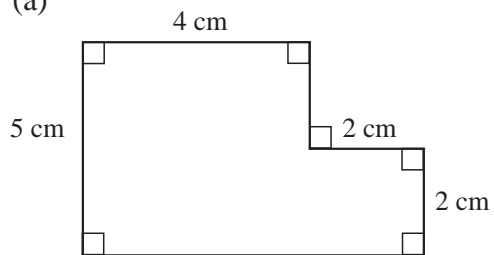
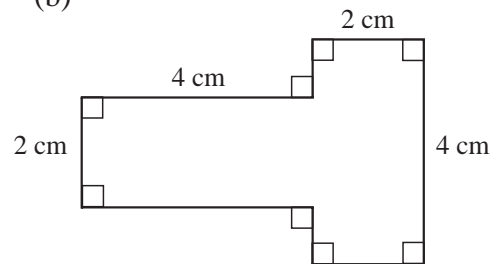1.   Find the area of these shapes.

      (a)
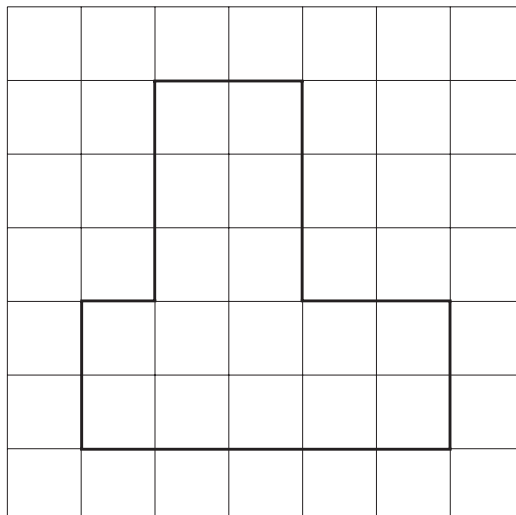
(b)

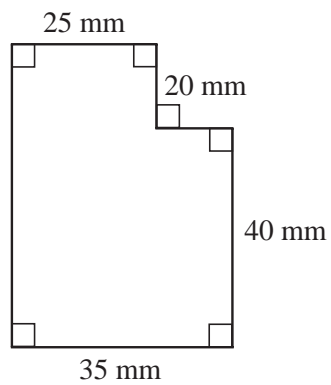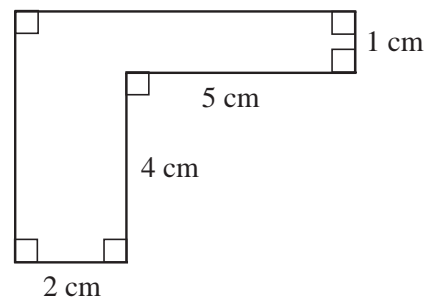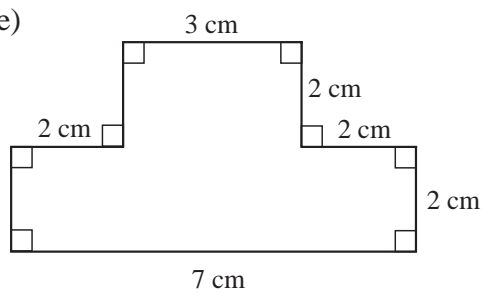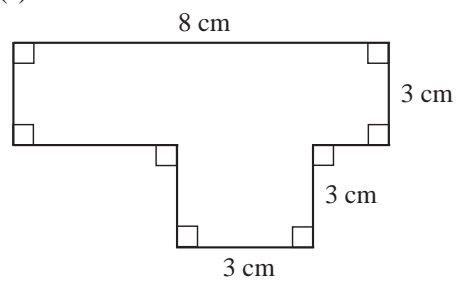2. Find the area of each of these shapes. The diagrams have not been drawn accurately.

(a)

4 cm

5 cm

2 cm

2 cm

(b)

2 cm

4 cm

2 cm

4 cm

(c)

25 mm

20 mm

40 mm

35 mm

(d)

1 cm

5 cm

4 cm

2 cm

(e)

3 cm

2 cm
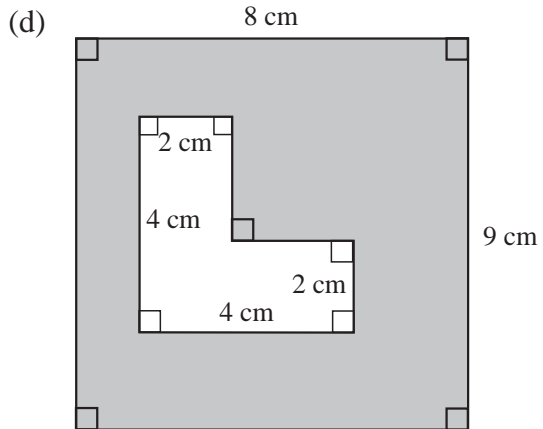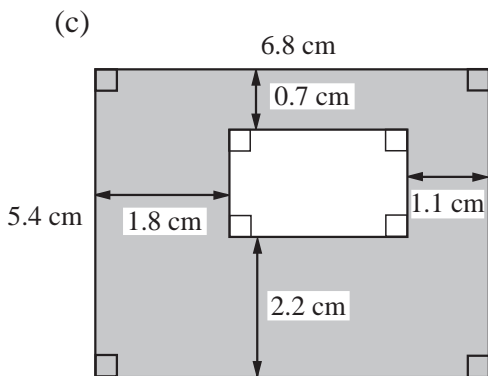
2 cm

2 cm

2 cm

7 cm

(f)

8 cm

3 cm

3 cm

3 cm

3.   Find the perimeter of the shapes in question 2.

4.   Find the area and perimeter of these shapes.

(a)

6 cm

3 cm   4 cm
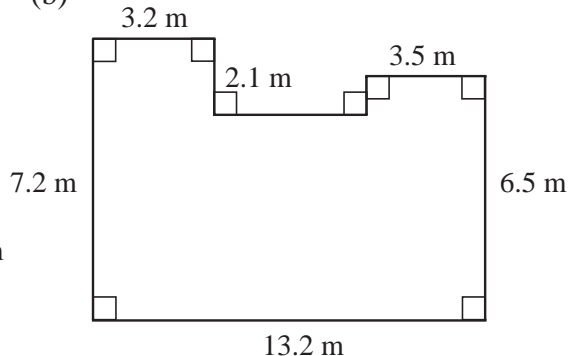
1 cm

(b)

2 mm

5 mm   1 mm   5 mm

2 mm   2 mm

(c)

2 cm

2 cm

2 cm

2 cm

2 cm

2 cm

6 cm

(d)

3 cm

3 cm

2 cm   6 cm

3 cm

3 cm

3 cm

10 cm
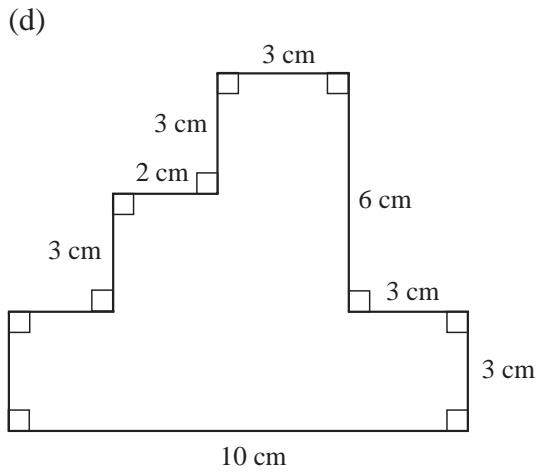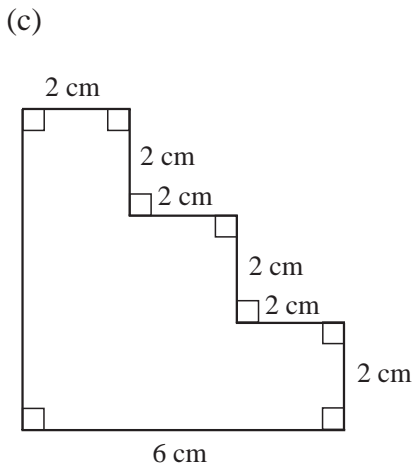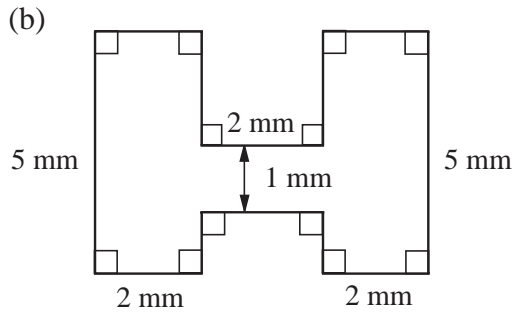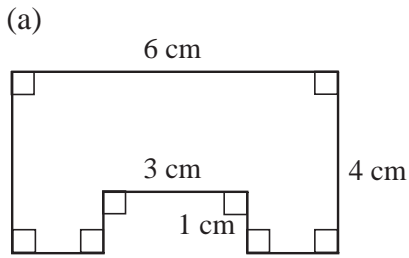
5.   Find the shaded area in each of the diagrams below.

(a)

4 cm

2 cm

2 cm   4 cm

(b)

8 cm

1 cm

1 cm   1 cm 6 cm

1 cm

(c)

6.8 cm

0.7 cm

5.4 cm   1.8 cm   1.1 cm

2.2 cm

(d)

8 cm

2 cm

4 cm   9 cm

2 cm

4 cm

6. Find the area and perimeter of these shapes.

    (a)

    3.1 cm

    3.4 cm

    3.8 cm

    3.7 cm

    (b)

    3.2 m

    3.5 m

    2.1 m

    7.2 m

    6.5 m

    13.2 m

    (c)

    5.6 cm

    5.8 cm   6.4 cm

    1.2 cm   0.8 cm

    (d)
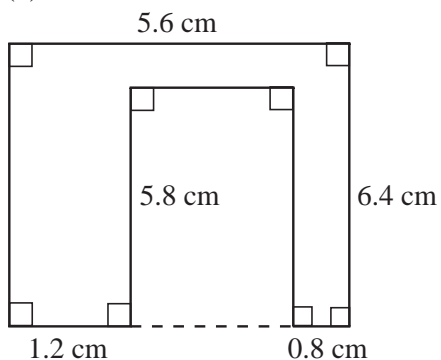
    2.2 cm
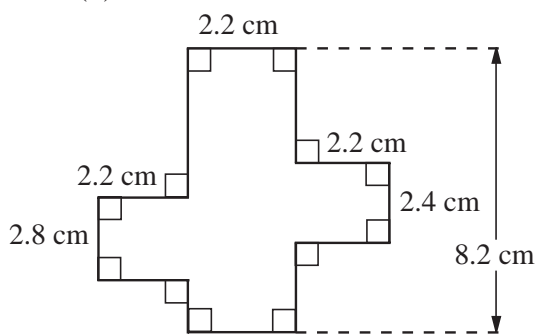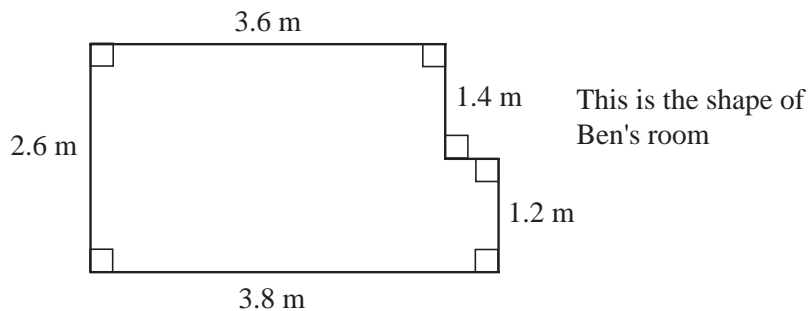
    2.2 cm

    2.2 cm

    2.4 cm

    2.8 cm

    8.2 cm
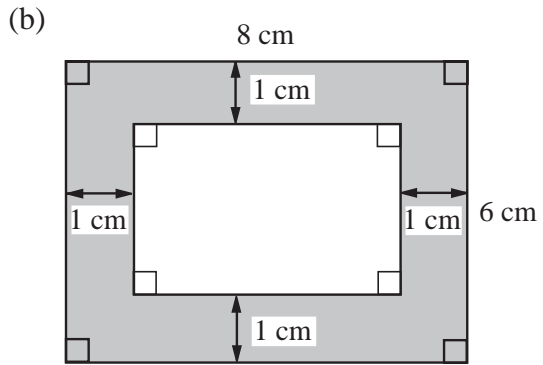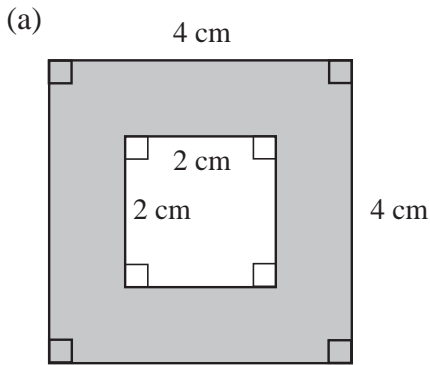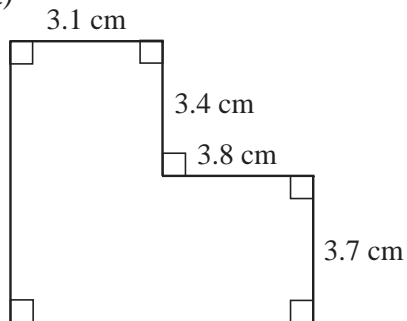
7. A lawn is 3 m by 5 m. A path 1 m wide is laid around the lawn.

    Find the area of the path.

    Path

    Lawn

    Path
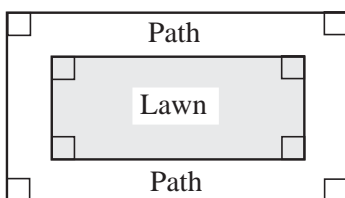
8. Ben's dad buys a carpet that is 3 m wide and 4 m long.

    3.6 m

    1.4 m   This is the shape of
               Ben's room

    2.6 m

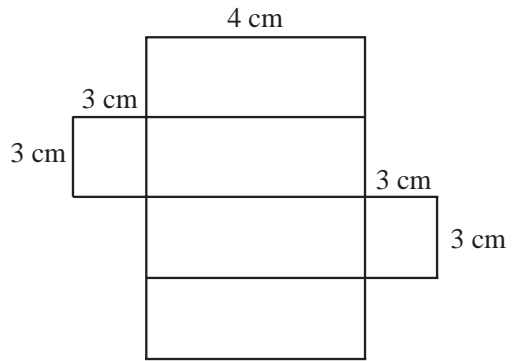   1.2 m

    3.8 m

    How much carpet is wasted?

9.   Wendy is going to paint the front of the house for her mum and dad.
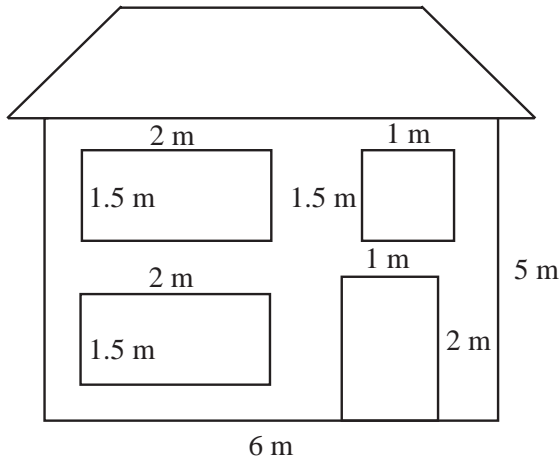
(a)   Find the area that needs to be painted.

(b)   Wendy gets 50p for every 1 m^2 that she paints. How much money does Wendy get?

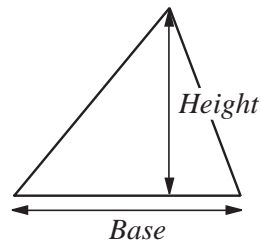10.   This shape can be cut out of card and folded to form a box.

      How much card is wasted if this shape is cut out of a sheet 15 cm by 20 cm?

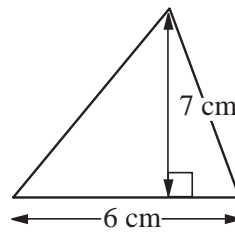# 9.5 The Area of a Triangle

For a triangle,

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$

Example 1

Find the area of the triangle shown.

Solution

$$\text{Area} = \frac{1}{2} \times 6 \times 7$$
$$= 21 \text{ cm}^2$$

Example 2

Find the area of the triangle shown.
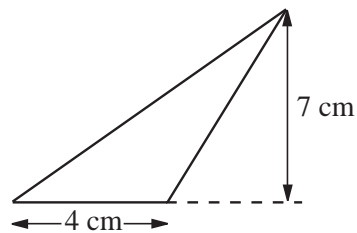
Solution

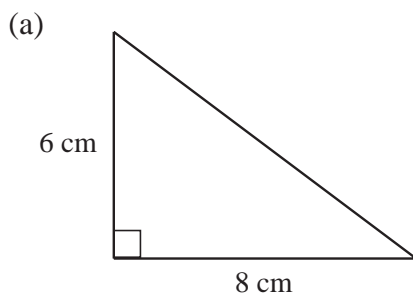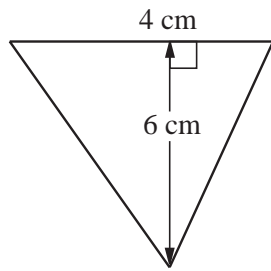$$\text{Area} = \frac{1}{2} \times 4 \times 7$$
$$= 14 \text{ cm}^2$$

Exercises

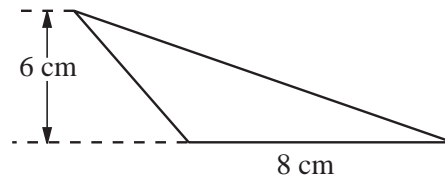1. Find the area of each of these triangles.

(a)

6 cm

8 cm
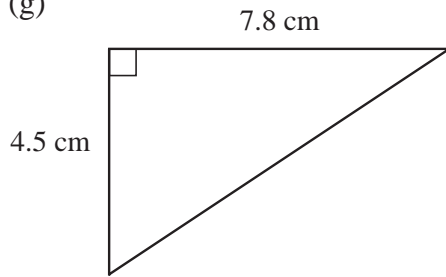
(b)

5 m

7 m

(c)

12 cm

8 cm

(d)

40 mm

30 mm

(e)

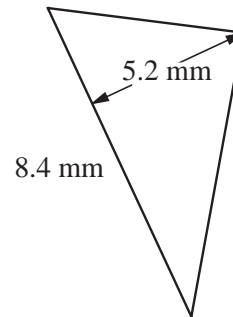4 cm
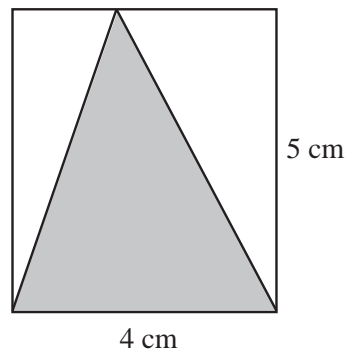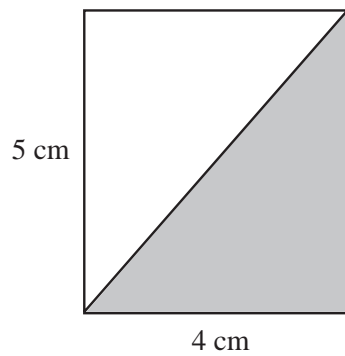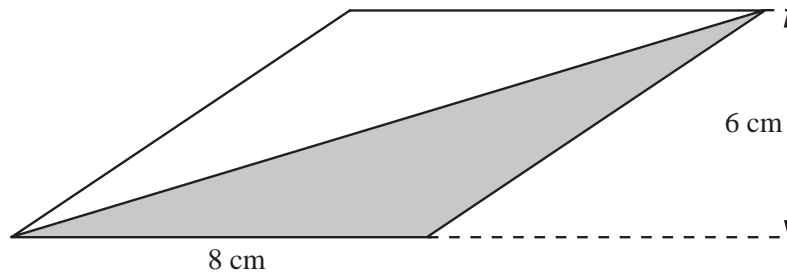
6 cm

(f)

6 cm

8 cm

(g)

7.8 cm

4.5 cm

(h)

5.2 mm

8.4 mm
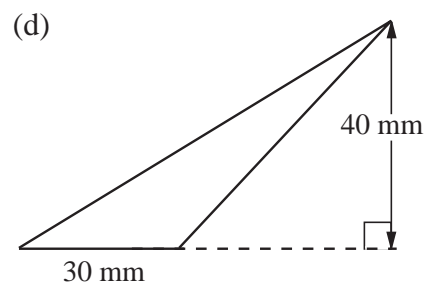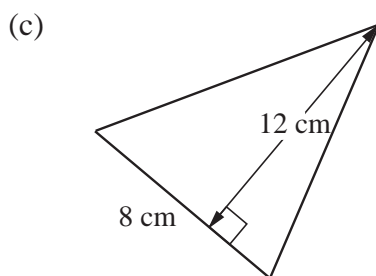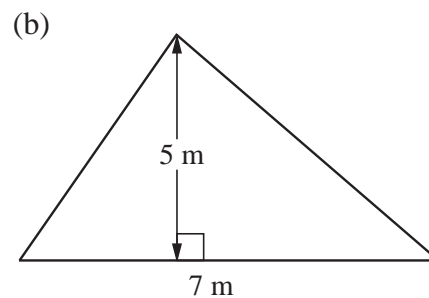
2. The diagrams show two triangles drawn inside a rectangle.

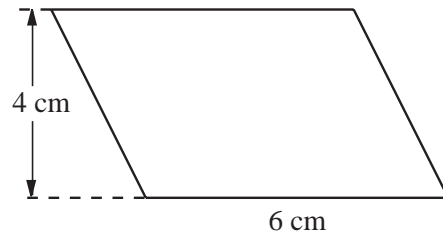Explain why the two shaded triangles have the same area.

5 cm

4 cm

5 cm

4 cm

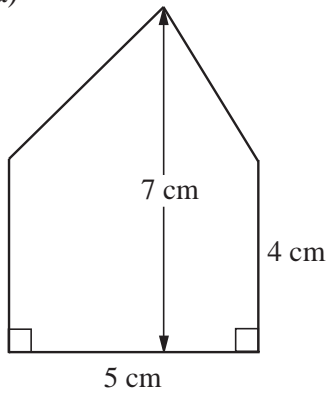3. (a) Find the area of the shaded triangle.

6 cm

8 cm

(b) What is the area of the parallelogram?

4. Find the area of this parallelogram.

4 cm

6 cm
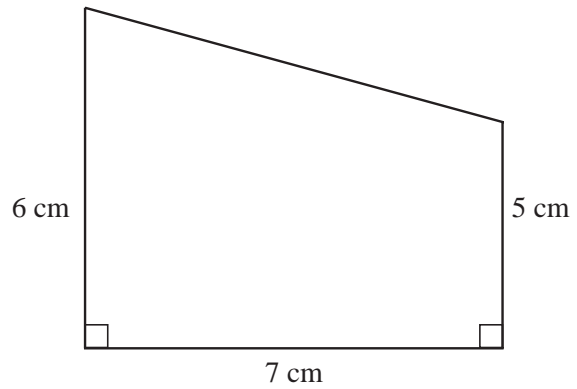
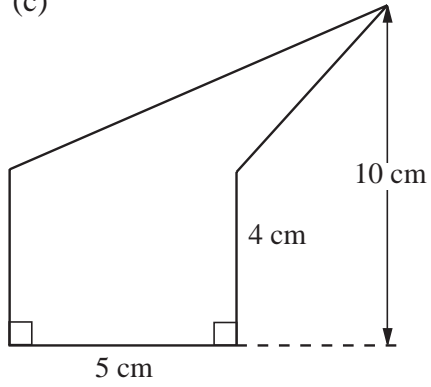5. Find the area of each of these shapes. They have not been drawn accurately.

(a)

7 cm

4 cm

5 cm

(b)

6 cm

5 cm

7 cm

(c)

10 cm

4 cm

5 cm

(d)

5 cm

1 cm

2 cm

5 cm
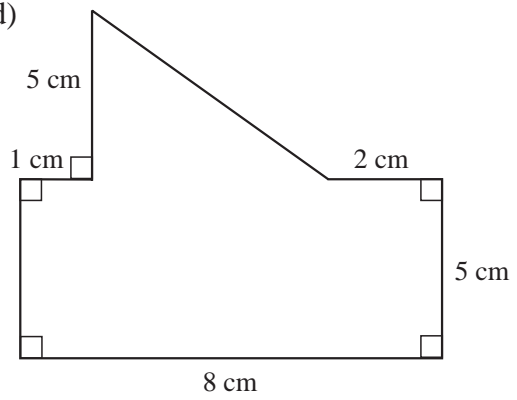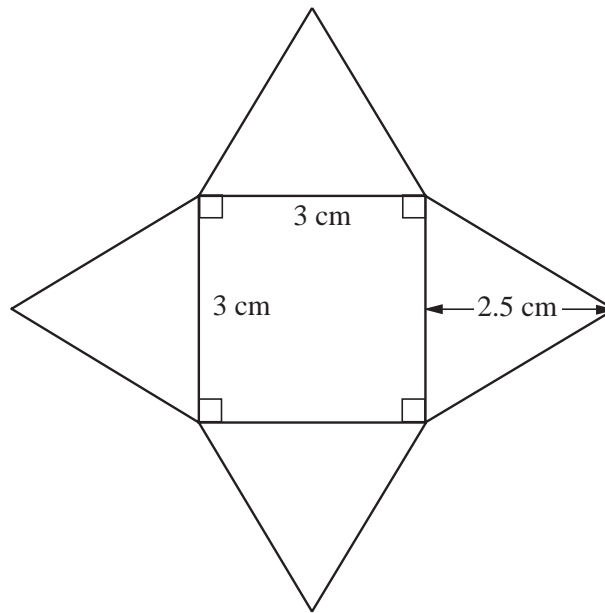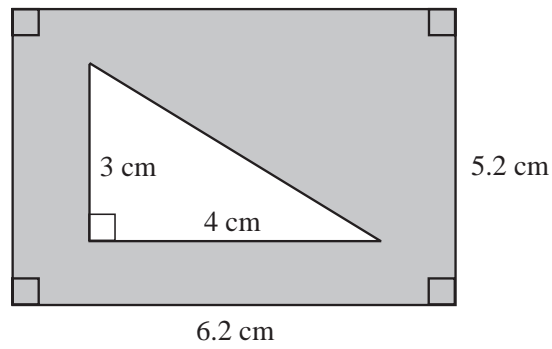
8 cm

6.   A pyramid can be made by folding up the shape below.

      What is the area of this shape?

3 cm

3 cm

2.5 cm

7.   Find the shaded areas.

      (a)

3 cm

4 cm

5.2 cm

6.2 cm

      (b)

2 cm

2 cm

6.2 cm

8 cm
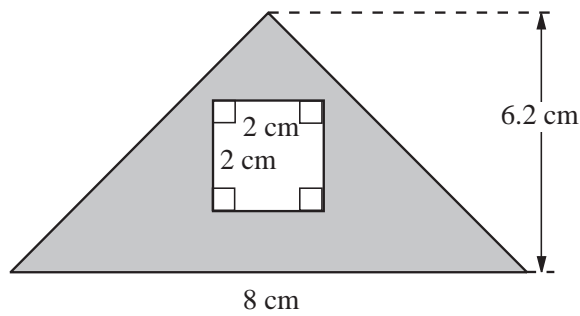
8.   Draw this triangle. Find its area to the nearest 0.1 cm^2.

8 cm
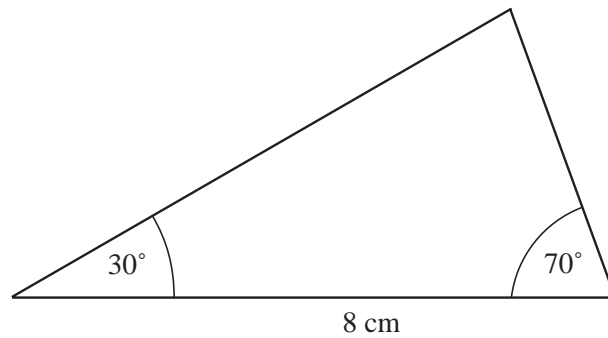
9.   Find the area of this triangle to the nearest 0.1 cm^2.

7 cm   6 cm

8 cm
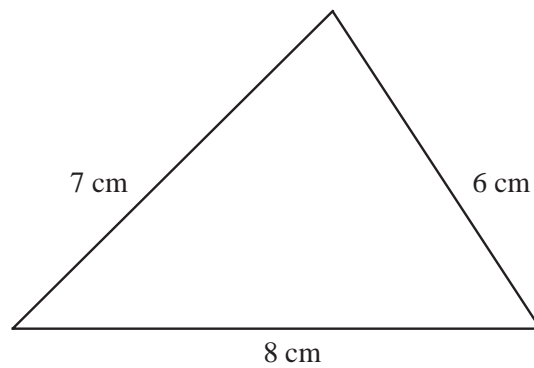
10.   Find the area of an equilateral triangle with sides of length 4 cm, giving your answer correct to 1 decimal place.

11.   What is the area of the biggest triangle that can be drawn inside a parallelogram with sides of length 10 cm and 12 cm?