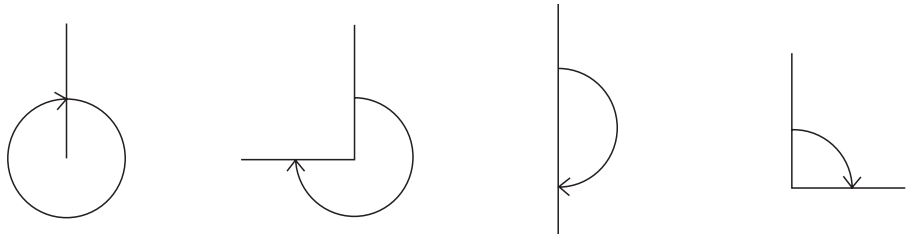
5 Angles

Angles are an important building block in geometry and trigonometry as you will see later. In this unit you will see how turns are related to angles, how to measure them and how to work out their size in particular problems.

5.1 Angles and Turns

You will need to understand clearly what the terms such as *turn*, *half turn*, etc. mean in terms of angles. There are $360\,°$ in one complete turn, so the following are true.



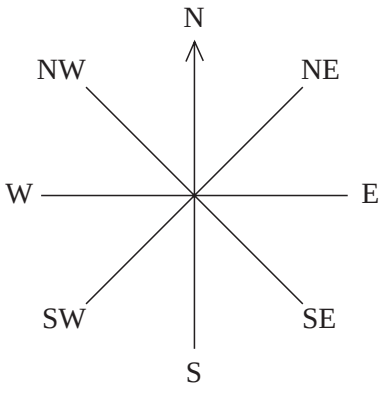
1 turn is $360\,°$

$\frac{3}{4}$ turn is $270\,°$

$\frac{1}{2}$ turn is $180\,°$

$\frac{1}{4}$ turn is $90\,°$

You also need to refer to compass points: (north (N), south (S), east (E), west (W), northeast (NE), southeast (SE), southwest (SW) and northwest (NW).



Turning from N to S is $180\,°$ clockwise or anticlockwise

Turning from NE to SE is $90\,°$ clockwise (or $270\,°$ anti-clockwise)

Turning clockwise from NE to E is $45\,°$ (or $315\,°$ anticlockwise)

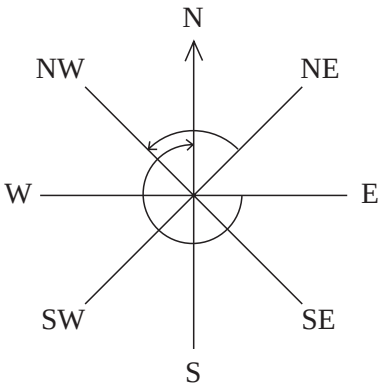
Example

What angle do you turn through if you turn

(a) from NE to NW *anticlockwise,*

(b) from E to N *clockwise*?

Solution

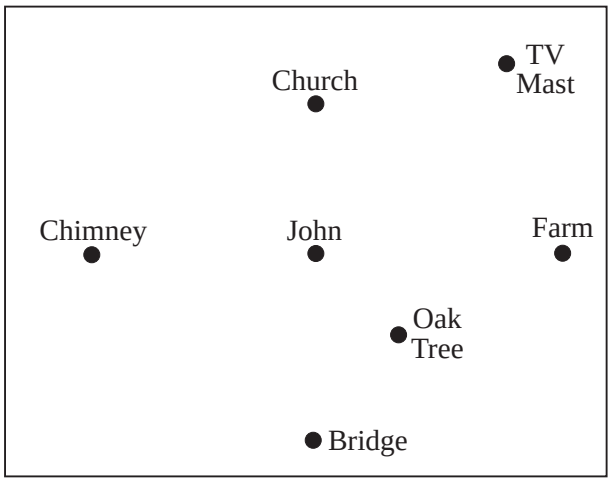


(a) You can see that this is $90\,°$ (or $\frac{1}{4}$ turn).

(b) This is a $\frac{3}{4}$ turn, i.e. $270\,°$.

Exercises

1. John is standing on a hill. The church is north of the point where he stands.



(a) In what direction is he facing if he looks at:

(i) the chimney, (ii) the bridge,

(iii) the TV mast, (iv) the farm,

(v) the oak tree?

(b) What angle does John turn through if he turns *clockwise* from looking at:

(i) the church to the farm,

(ii) the oak tree to the bridge,

(iii) the TV mast to the oak tree,

(iv) the bridge to the TV mast,

(v) the TV mast to the church?

(c) What would the angles be for question (b) if John turned *anticlockwise* instead of clockwise?

2. In a game, you spin a pointer and let it stop.

What angle does the pointer turn through if it completes:

(a) 1 turn, (b) 2 turns, (c) $\frac{3}{4}$ turn,

(d) $1\frac{1}{4}$ turns, (e) $1\frac{3}{4}$ turns, (f) $2\frac{1}{4}$ turns?

3. What angle do you turn through if you turn *clockwise* from facing:

(a) N to E, (b) W to NW, (c) SE to NW,

(d) NE to N, (e) W to NE, (f) S to SW,

(g) S to SE, (h) SE to SW, (i) E to SW?

4 What angle do you turn through if you turn *anticlockwise* from facing:

(a) N to SW, (b) S to SW,

(c) W to NW, (d) E to S?

5. In what direction will you be facing if you turn:

(a) 180 ° clockwise from NE,

(b) 180 ° anticlockwise from SE,

(c) 90 ° clockwise from SW,

(d) 45 ° clockwise from N,

(e) 225 ° clockwise from SW,

(f) 135 ° anticlockwise from N,

(g) 315° clockwise from SW?

6. Nigel stands on a low hill. The diagram on the next page shows some of the things he can see. Using information from the diagram, answer the following questions.

(a) What is NE of Nigel?

(b) What is SE of Nigel?

(c) Nigel turns from looking at the Old Fort to look at the ship. What angle does he turn through?

Explain why there is more than one answer to this question.

(d) What angle does Nigel turn through if:

(i) he turns clockwise from looking at the ship to the crane,

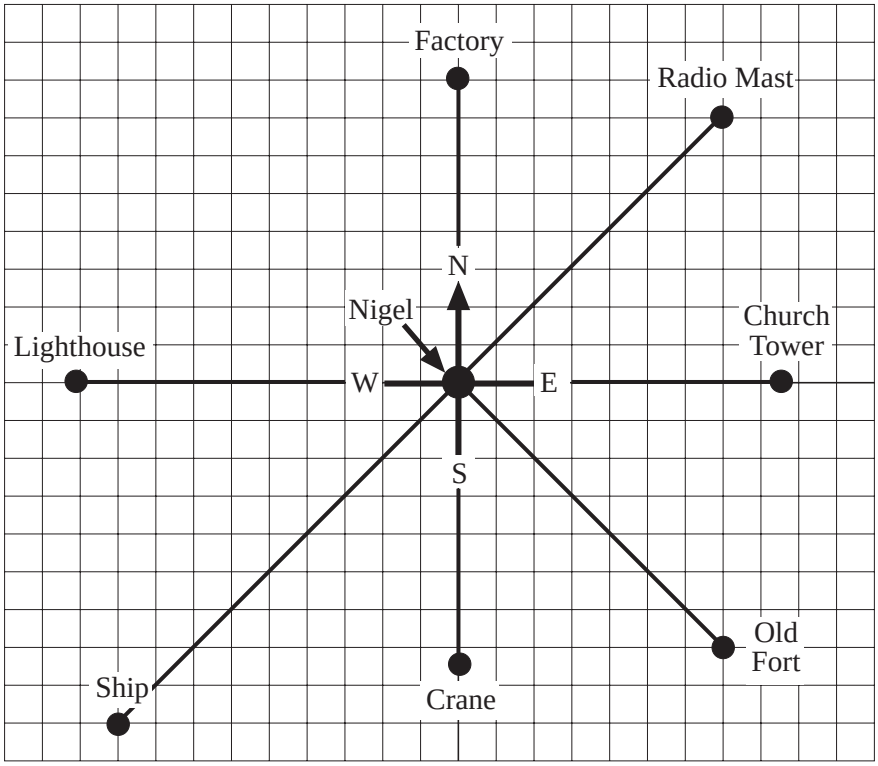
(ii) he turns anticlockwise from looking at the radio mast to the factory,

(iii) he turns anticlockwise from looking at the factory to the ship?

(e) Nigel starts looking at the factory. What does he end up looking at if he turns:

(i) 135 ° clockwise, (ii) 270 ° anticlockwise,

(iii) 225 ° clockwise, (iv) 405 ° clockwise?



7. Use the diagram on the next page to answer these questions.

(a) What is N of the shop?

(b) What is W of the church?

(c) What is E of the church?

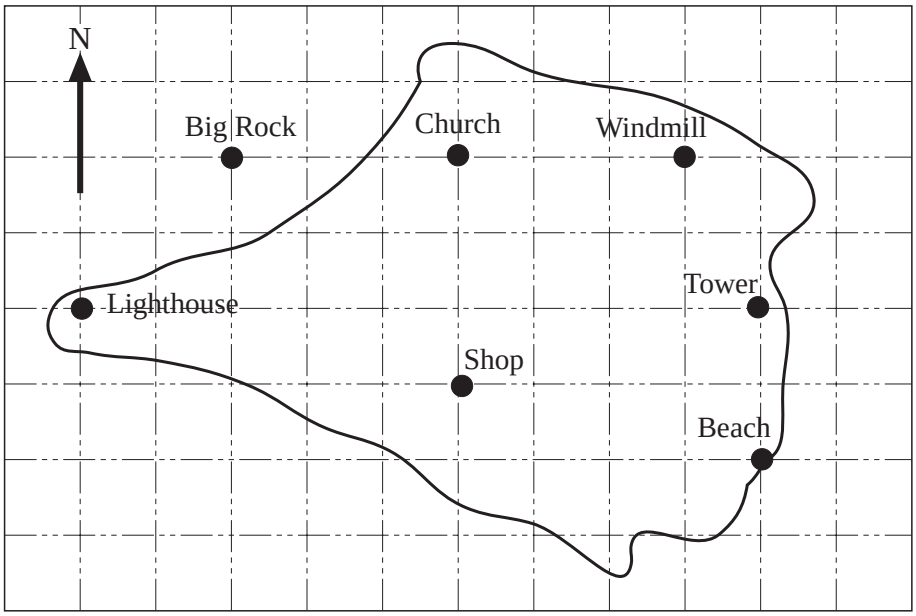
(d) What is E of Big Rock and NE of the shop?

(e) What is SW of Big Rock?

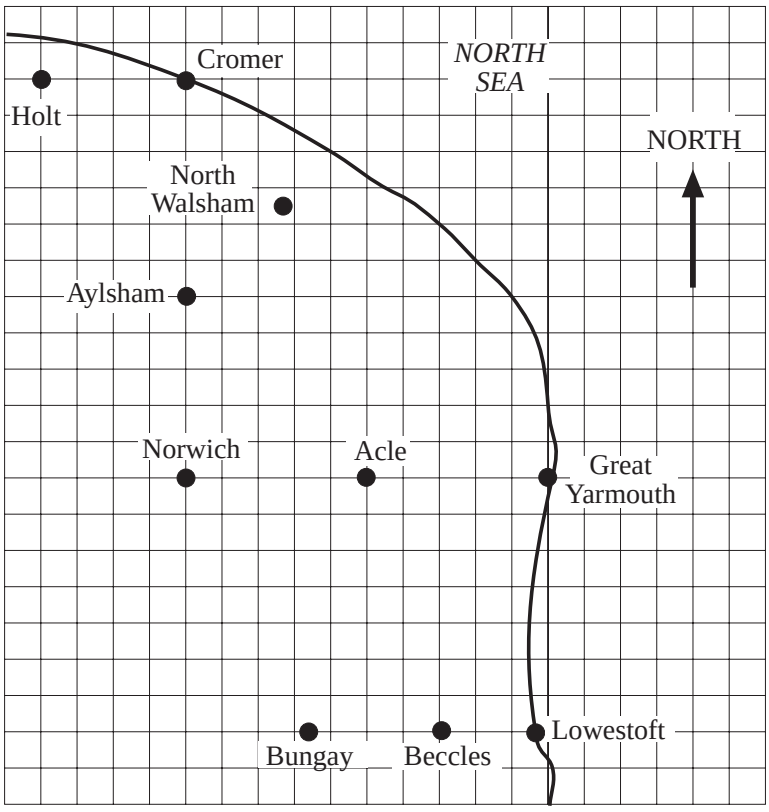
(f) In what direction should you walk from the beach to get to the tower?

(g) In what direction should you walk from the beach to get to the church?

(h) If you walk SE from the windmill, will you get to the tower? Explain your answer.



8. The map below shows part of East Anglia.



(a) What is east of Acle?

(b) What is north of Norwich and south of Cromer?

(c) What is SE of Norwich?

(d) What is NW of Acle?

(e) What is west of Cromer?

(f) What is west of Lowestoft and east of Bungay?

9. The sails of a windmill complete one full turn every 40 seconds.

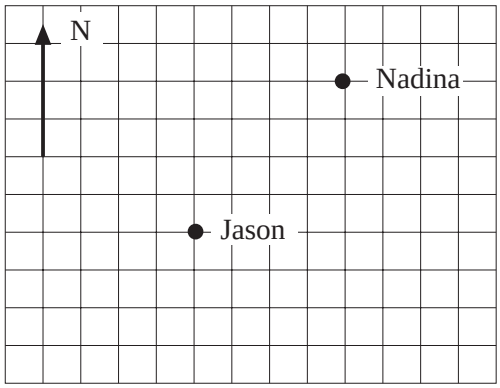
   (a) How long does it take the sails to turn through:

   (i) 180 ° (ii) 90 ° (iii) 45 °?

   (b) What angle do the sails turn through in:

   (i) 30 seconds, (ii) 15 seconds, (iii) 25 seconds?

10. The diagram shows the positions of Jason and Nadina. The arrow shows the direction of north.



   (a) Copy or trace the diagram.

   (b) Karen is west of Nadina and north of Jason. Mark Karen's position on your diagram.

   (c) Jenny is east of Jason and southeast of Nadina. Mark Jenny's position on your diagram.

   (d) Wendy is west of Jenny and southeast of Karen. Where is Wendy in relation to Nadina?

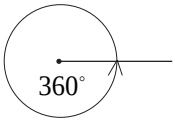
   (e) Jai is north of Jason and south of Karen. Describe where he could be standing.

5.2 Measuring Angles

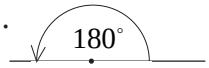
A *protractor* can be used to measure or draw angles.

Note

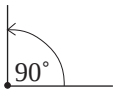
The angle around a complete circle is is $360\,°$.



The angle around a point on a straight line is $180\,°$.

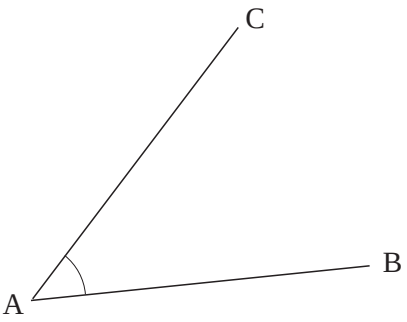


A right angle is $90\,°$.



Example 1

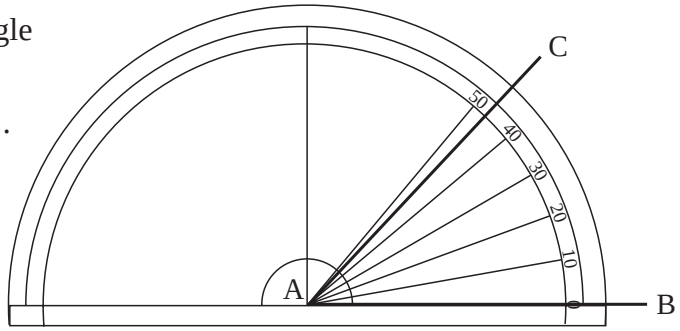
Measure the angle CAB in the triangle shown.



Solution

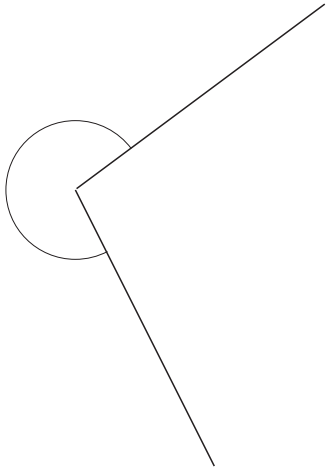
Place a protractor on the triangle as shown.

The angle is measured as $47\,°$.



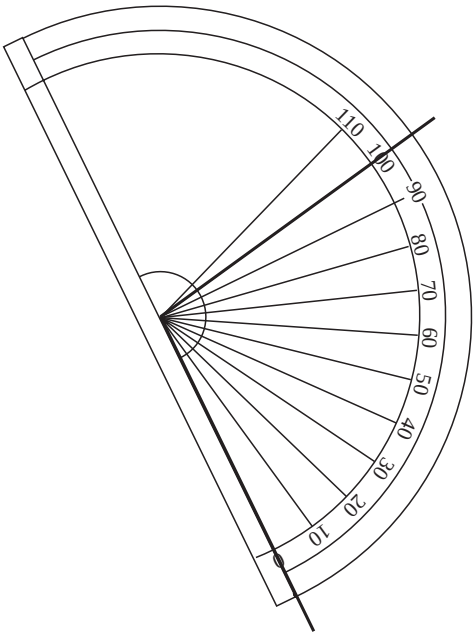
Example 2

Measure this angle.



Solution

Using a protractor, the smaller angle is measured as 100 °.



So

required angle = 360 ° − 100 °
= 260 °

Example 3

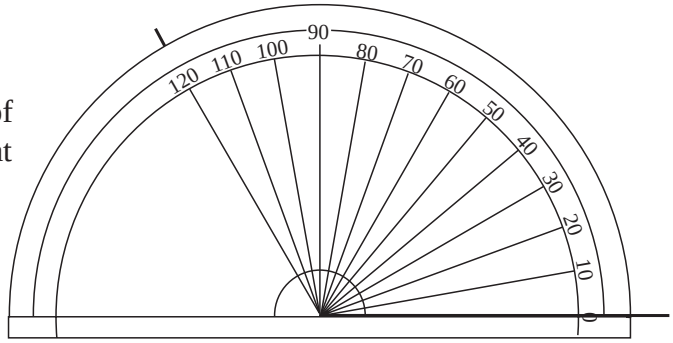
Draw angles of

(a) 120 ° (b) 330 °.

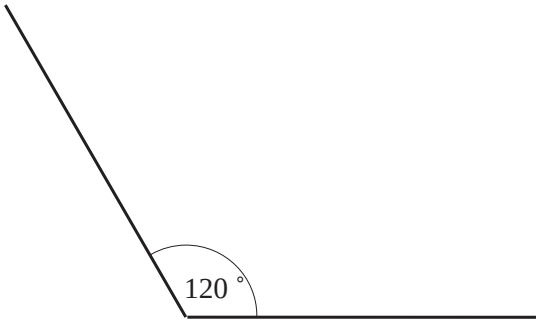
Solution

(a) Draw a horizontal line.

Place a protractor on top of the line and draw a mark at 120 °.



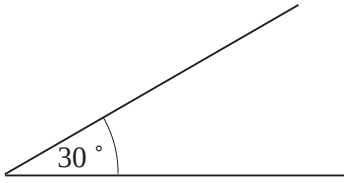
Then remove the protractor and draw the angle.



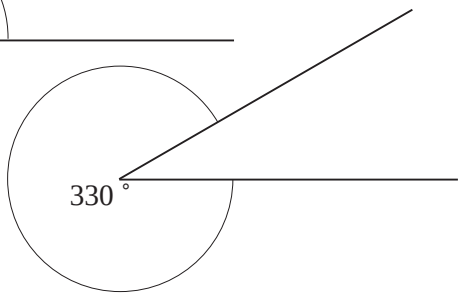
(b) To draw the angle of 330 °, first subtract 330 ° from 360 °:

360 ° − 330 ° = 30 °

Draw an angle of 30 °.



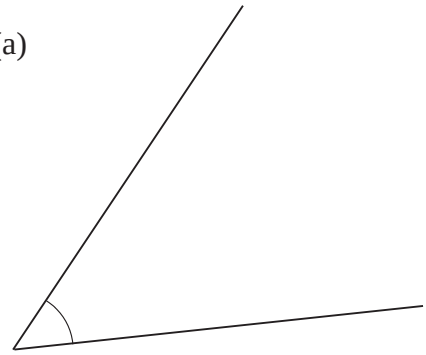
The larger angle will be 330 °.



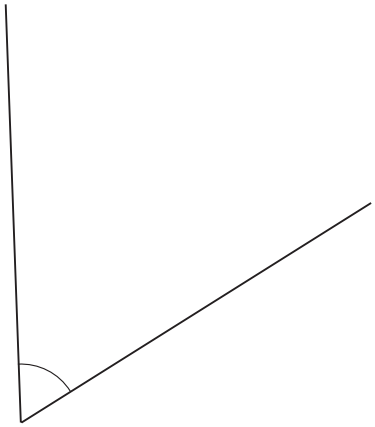
Exercises

1. For each of the following angles, first estimate the size of the angle and then measure the angle to see how good your estimate was.

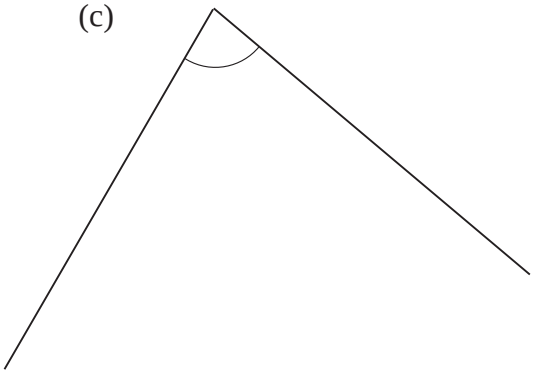
(a)



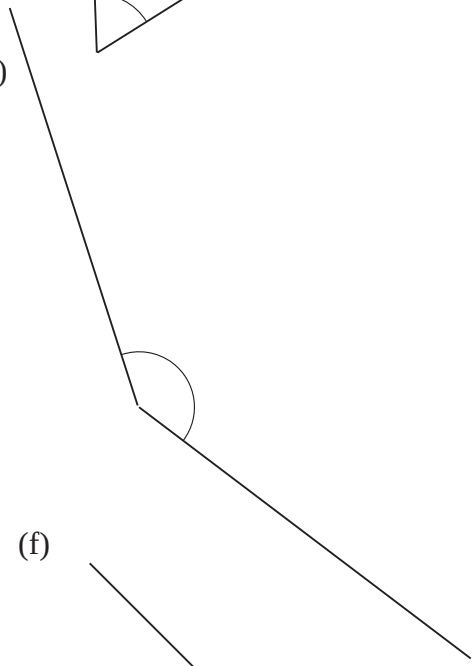
(b)



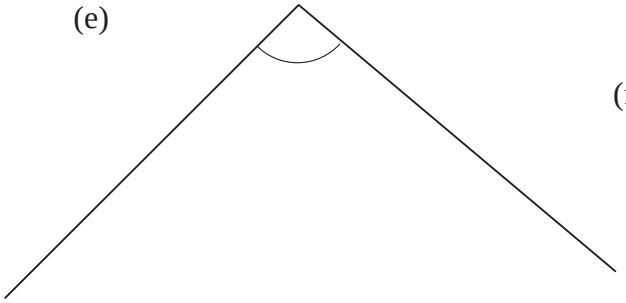
(c)



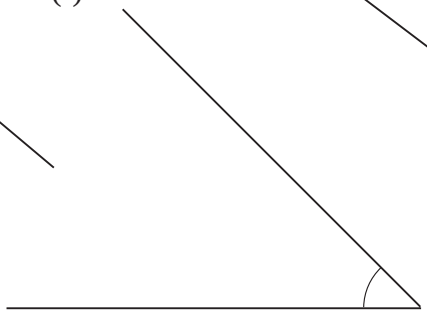
(d)



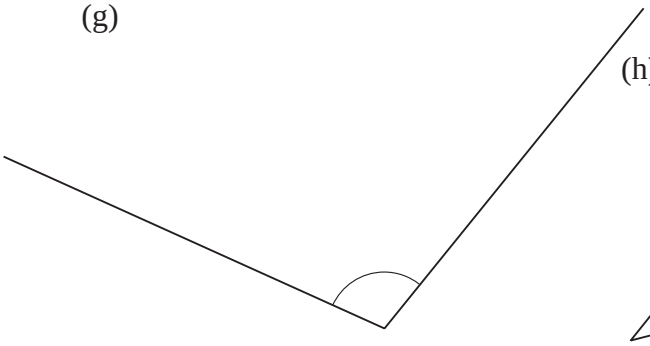
(e)



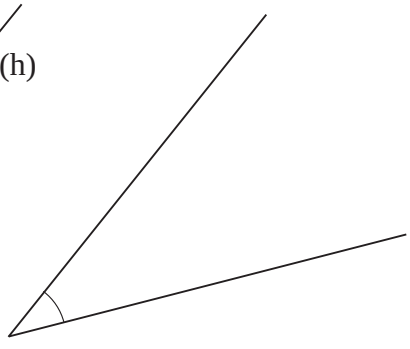
(f)



(g)

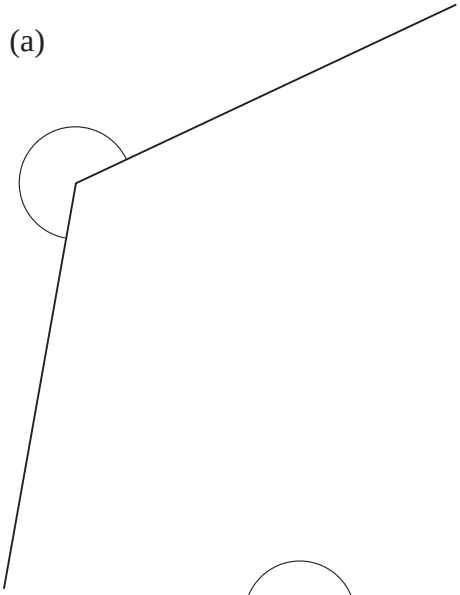


(h)

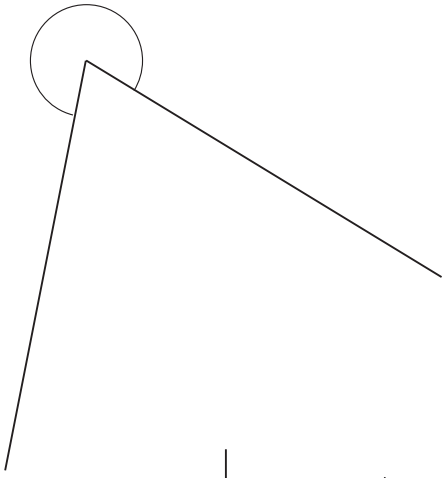


2. Estimate and measure the size of each of these reflex angles.

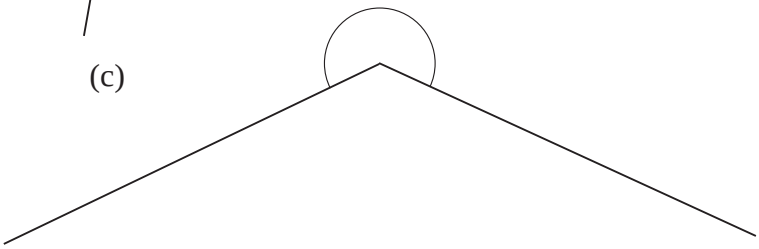
(a)



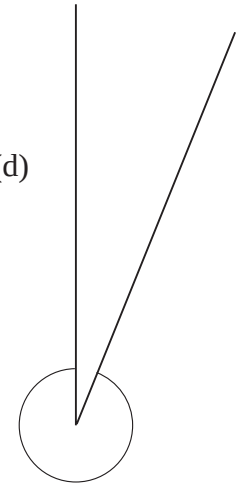
(b)



(c)

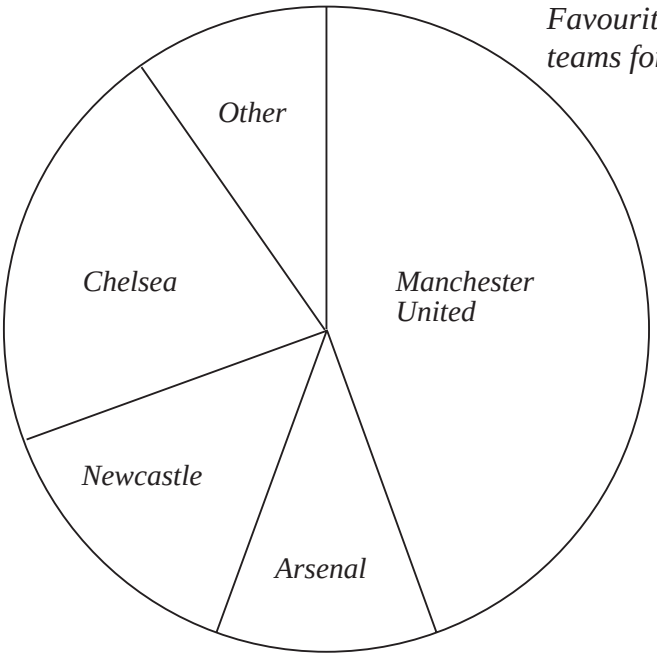


(d)



3. (a) Measure each of the angles in this pie chart.

Favourite football teams for class 7G



(b) Explain how you can tell that Manchester United is the most popular of these teams.

(c) Which is the second most popular team?

4. Draw the following angles:

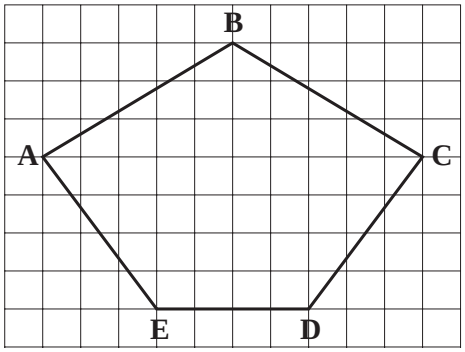
(a) 20 ° (b) 42 ° (c) 80 ° (d) 105 °

(e) 170 ° (f) 200 ° (g) 275 ° (h) 305°

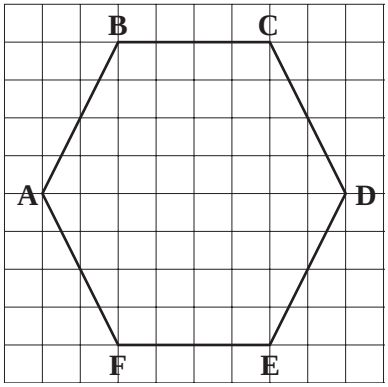
5. In which of these polygons are the angles all the same size?

Find all the angles in each polygon. *(You may need to copy the shapes into your book and extend the lines.)*

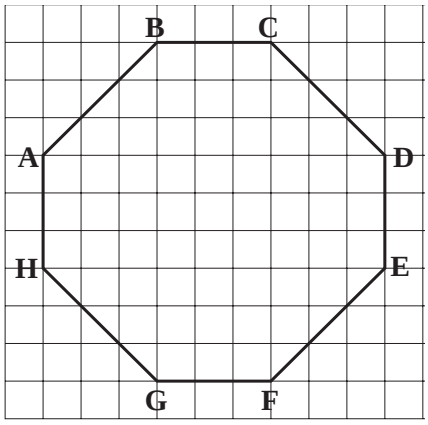
(a)



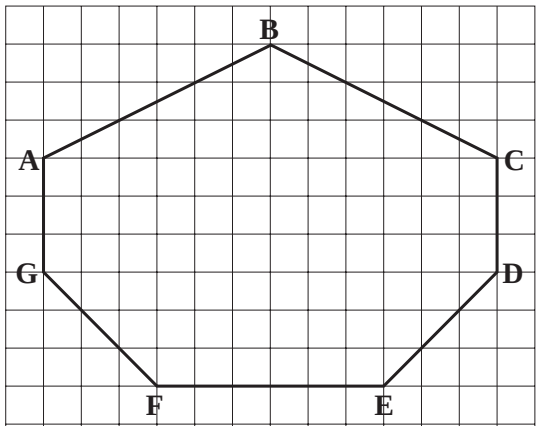
(b)



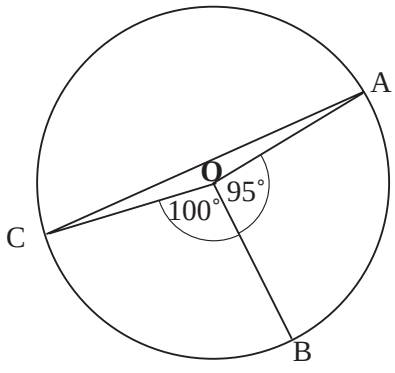
(c)



(d)



6. (a) Draw the shape below, where O is the centre of the circle. Make the radius of your circle 6 cm.



(b) Measure the distances between AB, BC and AC.

7. Ravinder finds out the favourite sports for members of his class. He works out the angles in the list shown opposite for a pie chart.

Draw the pie chart.

Sport	*Angle*
Football	110 °
Swimming	70 °
Tennis	80 °
Rugby	40 °
Hockey	30 °
Badminton	10 °
Other	20 °

5.3 Classifying Angles

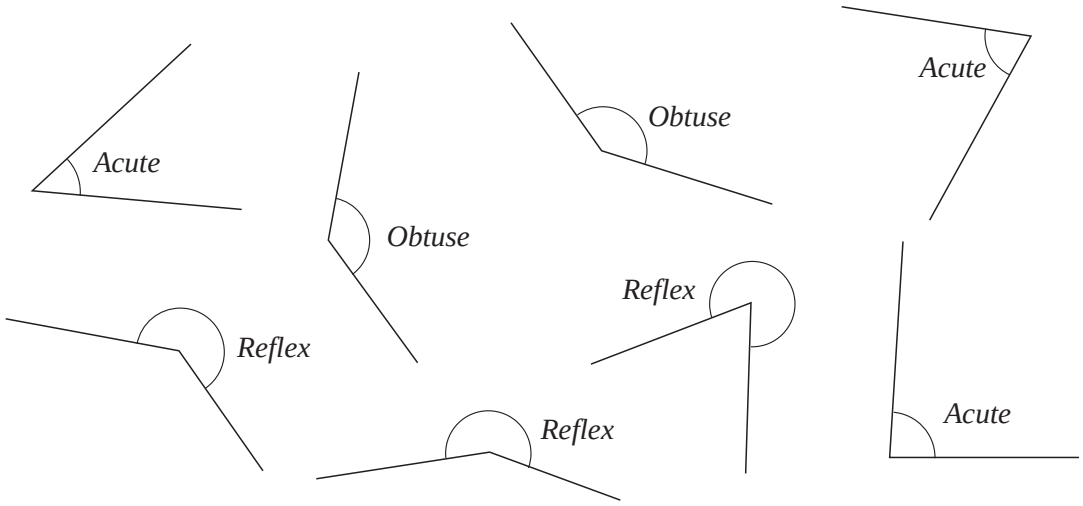
Angles of less than 90 ° are *acute* angles

Angles between 90 ° and 180 ° are *obtuse* angles

Angles between 180 ° and 360 ° are *reflex* angles

So you can easily identify the three types of angle.

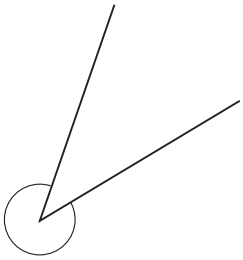
Here are some examples.



Exercises

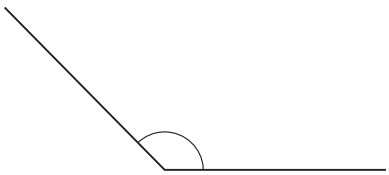
1. Is each angle below *acute*, *obtuse* or *reflex*?

(a)

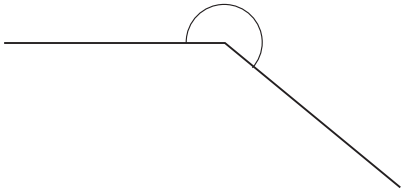


(b)

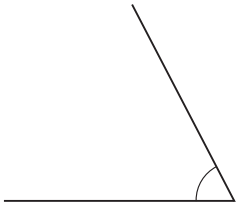
(c)



(d)



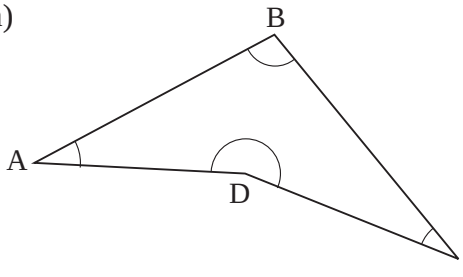
(e)



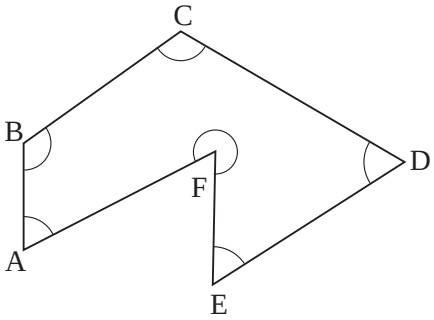
(f)

2. For each shape below state whether the angle at each corner is *acute*, *obtuse* or *reflex*.

(a)



(b)



3. (a) Draw a triangle with *one* obtuse angle.

   (b) Draw a triangle with *no* obtuse angles.

4. Draw a four-sided shape with:

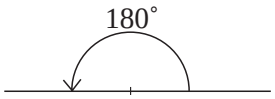
   (a) one *reflex* angle,

   (b) two *obtuse* angles.

5.4 Angles on a Line and Angles at a Point

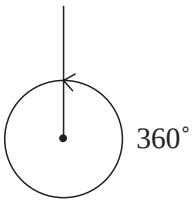
Remember that

(a) angles on a line add up to $180\,^\circ$



and

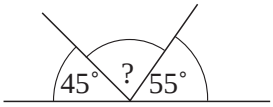
(b) angles at a point add up to $360\,^\circ$.



These are two important results which help when finding the size of unknown angles.

Example 1

What is size of the angle marked ?



Solution

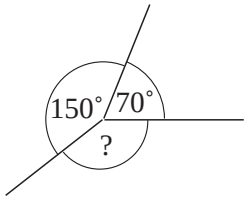
$$45\,^\circ + 55\,^\circ = 100\,^\circ$$

So $\text{angle} = 180\,^\circ - 100\,^\circ$

$= 80\,^\circ$

Example 2

What is the size of the angle marked ?



Solution

$$70\,^\circ + 150\,^\circ = 220\,^\circ$$

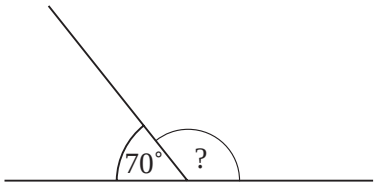
So $\text{angle} = 360\,^\circ - 220\,^\circ$

$= 140\,^\circ$

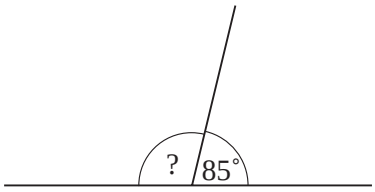
Exercises

1. Calculate the unknown angle in each of the following diagrams.

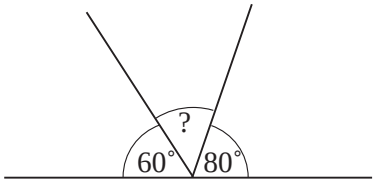
(a)



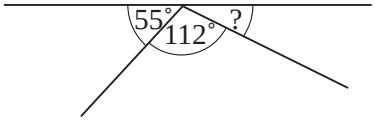
(b)



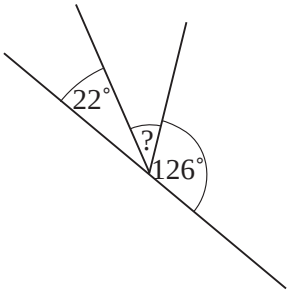
(c)



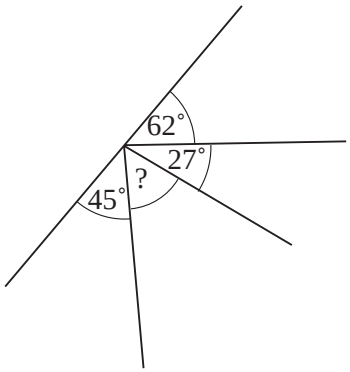
(d)



(e)

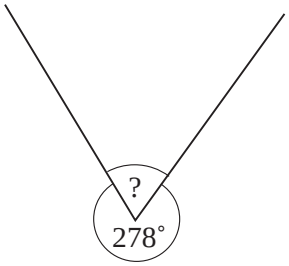


(f)

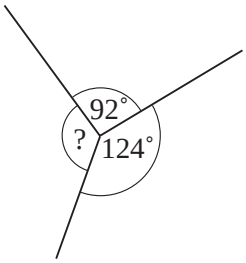


2. Calculate the unknown angle in each diagram.

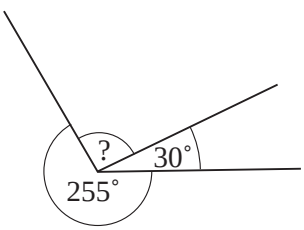
(a)



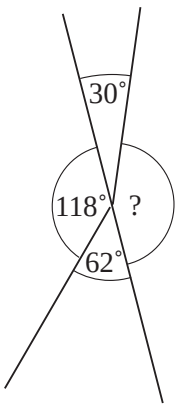
(b)



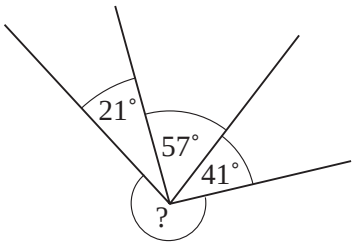
(c)



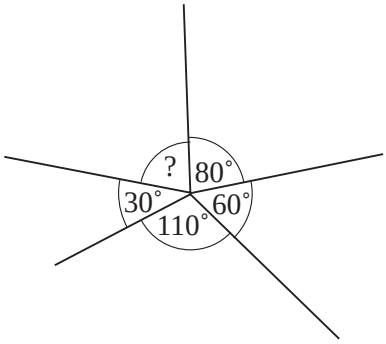
(d)



(e)

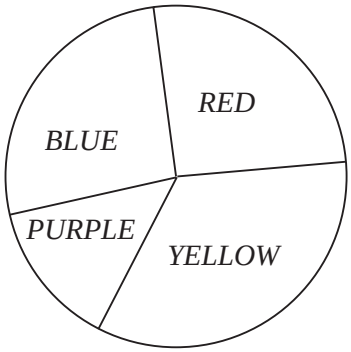


(f)



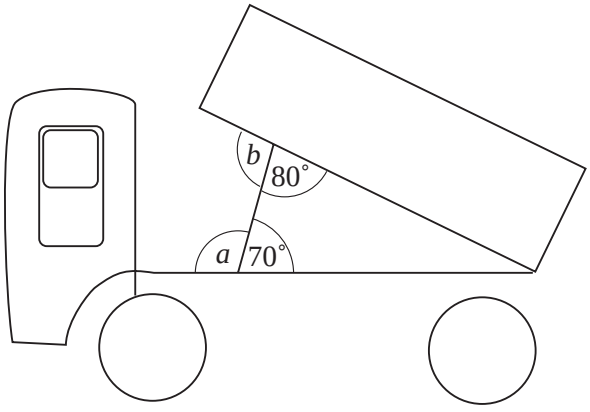
3. Some of the angles in the pie chart have been calculated:

Red	90 °
Blue	95 °
Purple	50 °



What is the angle for *yellow*?

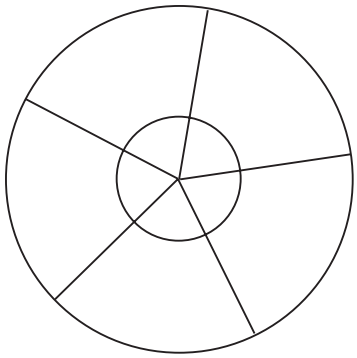
4. The picture shows a tipper truck.



(a) Find the angles marked *a* and *b*.

(b) The 80 ° angle decreases to 75 ° as the tipper tips further. What happens to angle *b*?

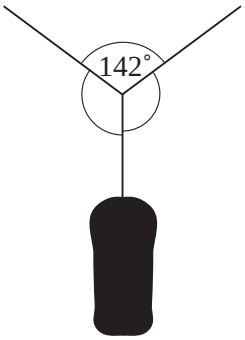
5. The diagram shows a playground roundabout viewed from above. Five metal bars are fixed to the centre of the roundabout as shown. The angles between the bars are all the same size.



(a) What size are the angles?

(b) What size would the angles be if there were 9 metal bars instead of 5?

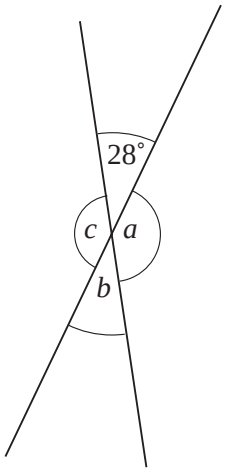
6.



A boy hangs a punchbag on a washing line.

Find the unknown angles if both angles are the same size.

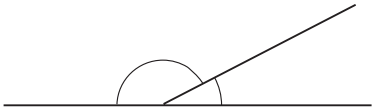
7. The diagram shows two straight lines.



Find the angles *a*, *b* and *c*.

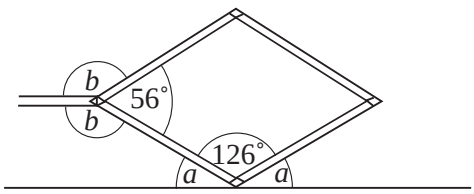
What do you notice?

8. In the diagram the large angle is 4 times bigger than the smaller angle.



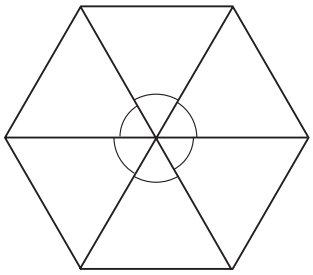
Find the two angles.

9. The picture shown a jack, that can be used to lift up a car.



Find the angles marked *a* and *b*.

10. The diagram shows a regular hexagon.



(a) Find the size of each of the angles marked at the centres of the hexagon.

(b) What would these angles be if the polygon was a decagon (10 sides).

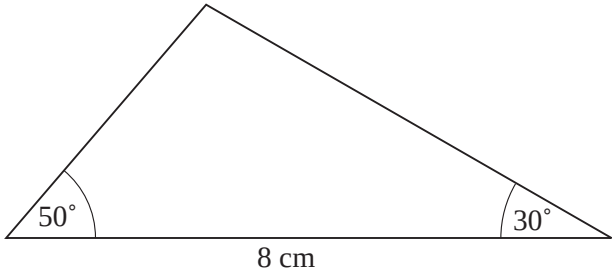
(c) If the angles were $30°$, how many sides would the polygon have?

5.5 Constructing Triangles

Here you will see how to construct triangles.

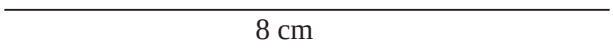
Example 1

Draw this triangle and measure the unknown angle.

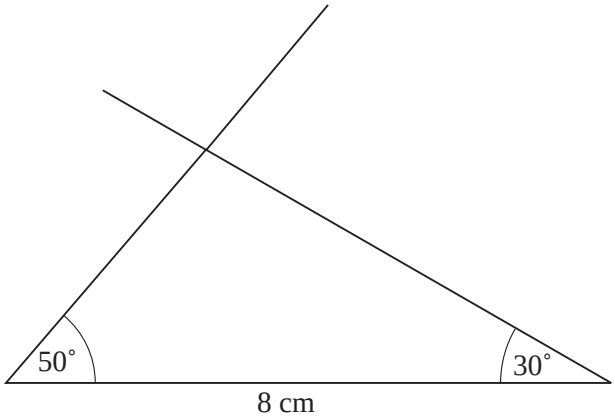


Solution

First draw the base line of 8 cm.



At each end, use a protractor to draw lines at angles $50°$ and $30°$ to the line.

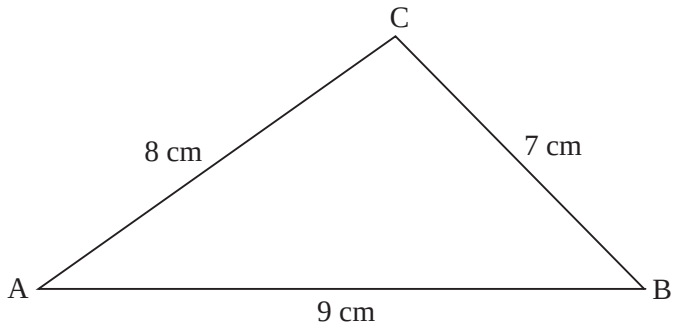


The intersection of these two lines is the third point of the triangle.

This angle measures about $100°$.

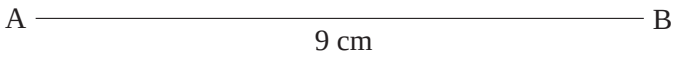
Example 2

Draw this triangle.

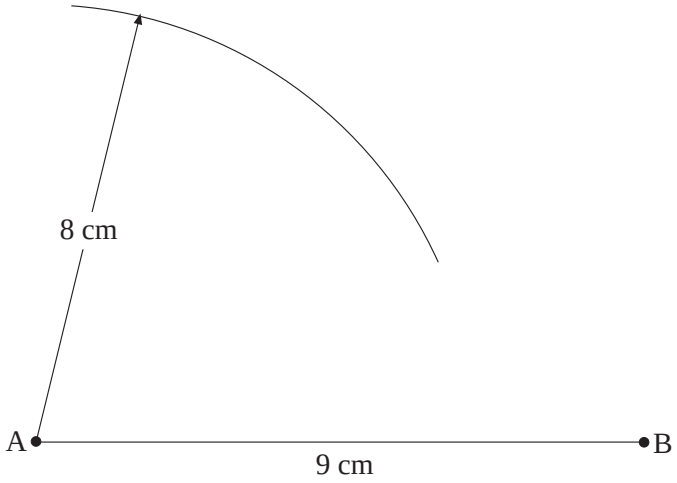


Solution

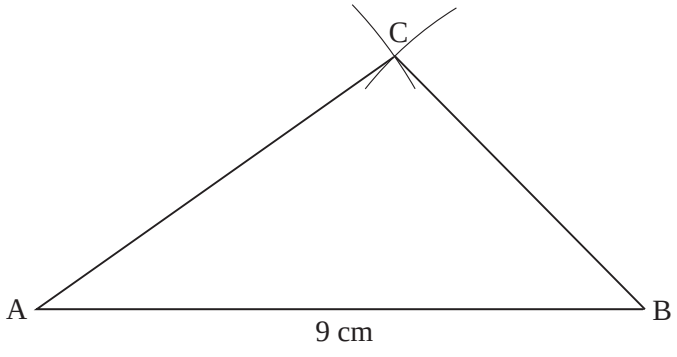
First draw the base line, AB, of length 9 cm.



Then set your compass so that the pencil tip is 8 cm from the point and draw an arc with its centre at A, as shown,



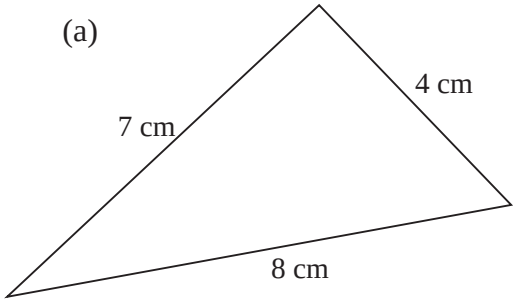
Then draw a similar arc with your compass set at 7 cm and B as the centre. The point where the two arcs cross is the third corner of the triangle.



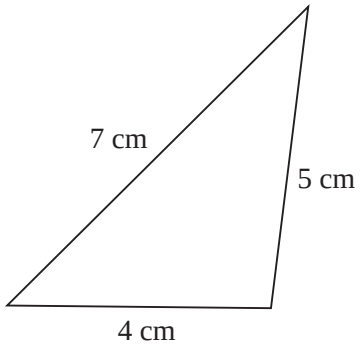
Exercises

1. Draw these triangles accurately. In each triangle, measure the angles and find their total.

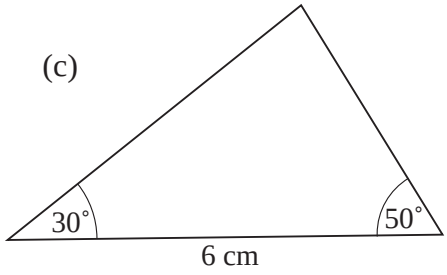
(a)



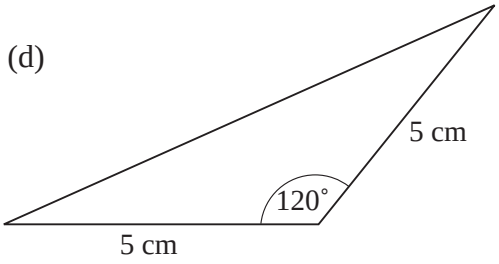
(b)



(c)



(d)

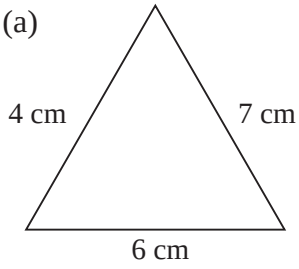


2. Compare your triangles with those drawn by other people in your class. Do your triangles look the same?

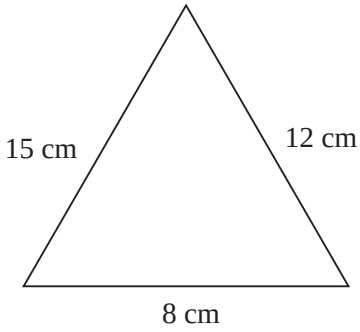
3. Explain why you cannot draw a triangle with sides of lengths 12 cm, 5 cm and 4 cm.

4. Which of these triangles can you draw?

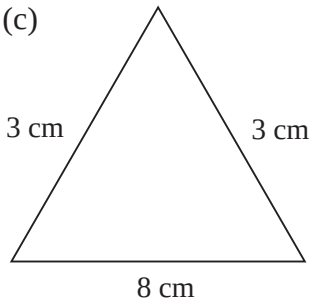
(a)



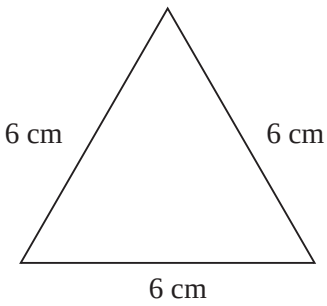
(b)



(c)

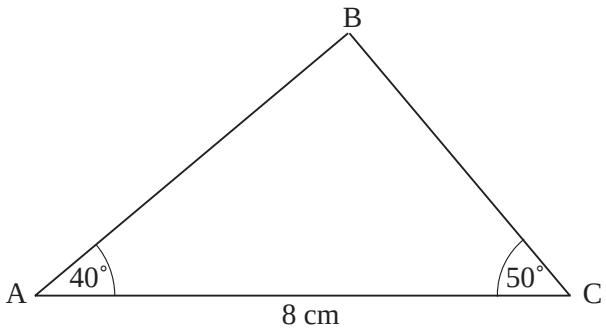


(d)



Draw those that are possible and measure the angles in them.

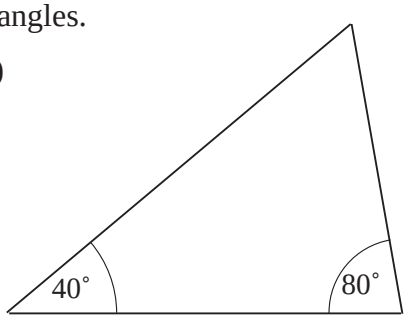
5. (a) Draw the triangle below and measure the lengths of the two sloping sides of the triangle.



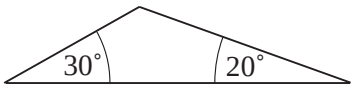
(b) Measure the third angle in the triangle.

6. Draw each triangle below and measure the third angle in each of the triangles.

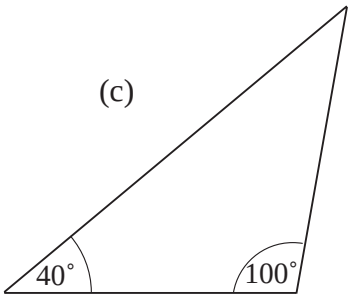
(a)



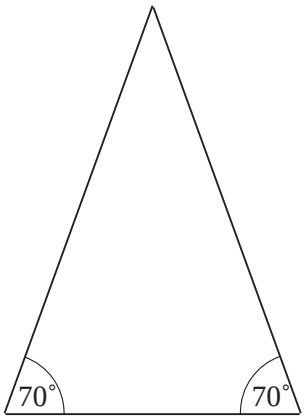
(b)



(c)



(d)

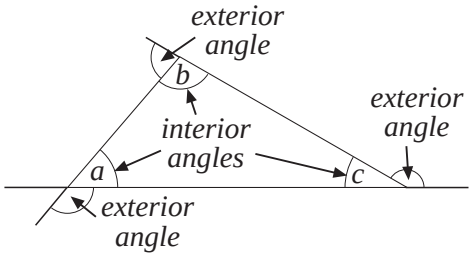


What do you notice?

5.6 Finding Angles in Triangles

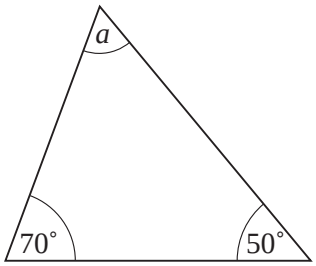
The interior angles of any triangle will always sum (add up) to 180 °.

$$a + b + c = 180\ °$$



Example

Find the angle marked *a* in the diagram opposite.



Solution

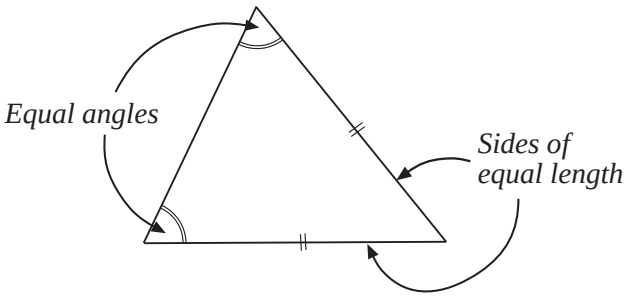
$70\ ° + 50\ ° = 120\ °$

So $180\ ° - 120\ ° = 60\ °$

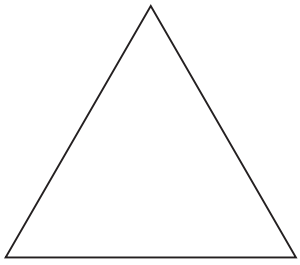
and $a = 60\ °$

The final part of this section deals with the classification of triangles.

ISOSCELES TRIANGLE



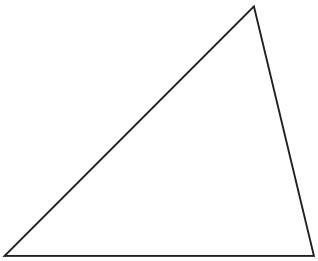
EQUILATERAL TRIANGLE



All sides are the same length

All angles are 60°

SCALENE TRIANGLE



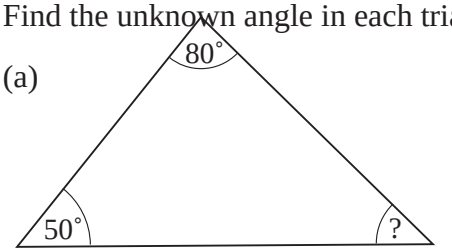
All sides have different lengths.

All angles are of different sizes.

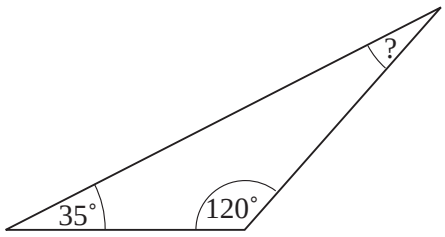
Exercises

1. Find the unknown angle in each triangle.

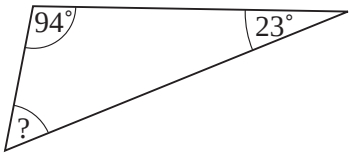
(a)



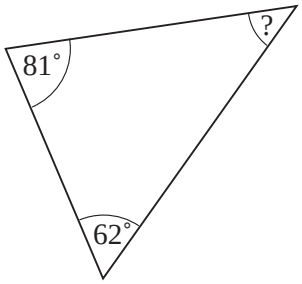
(b)



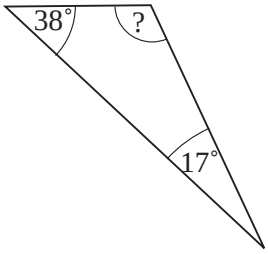
(c)



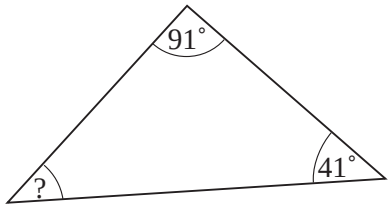
(d)



(e)

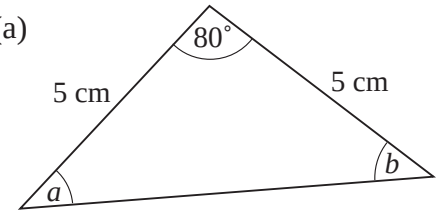


(f)

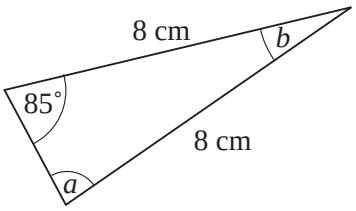


2. Find the unknown angles in each of the following triangles.

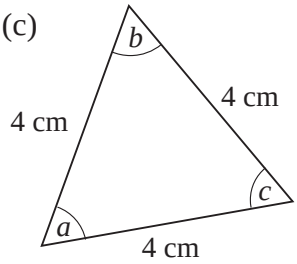
(a)



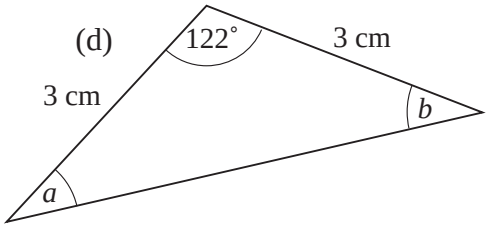
(b)



(c)

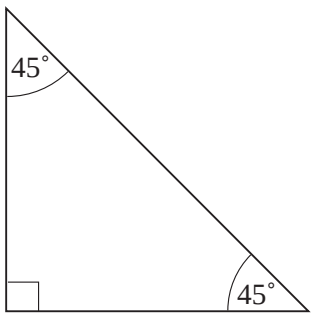


(d)

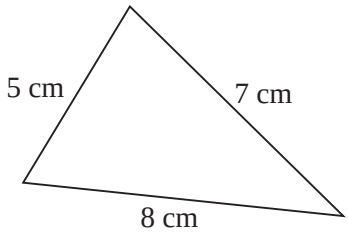


3. State whether each triangle below is *isosceles, equilateral* or *scalene.*

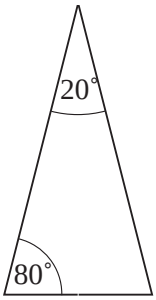
(a)



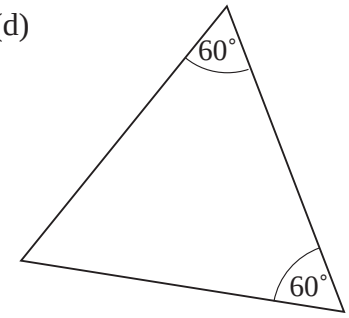
(b)



(c)

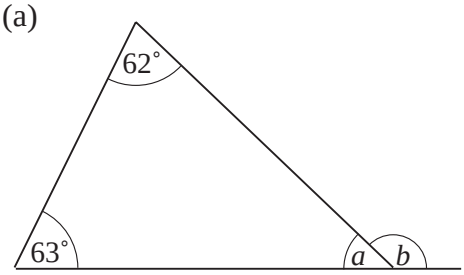


(d)

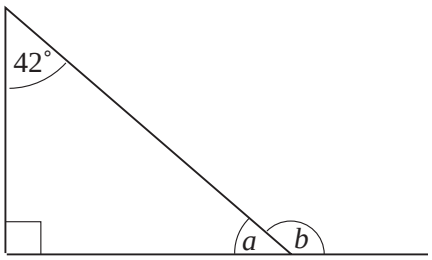


4. For each triangle below, find the unknown *interior* angle and the marked *exterior* angle.

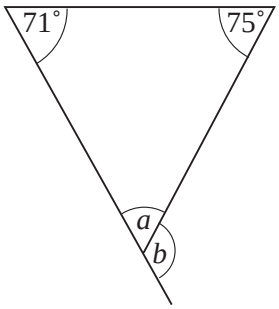
(a)



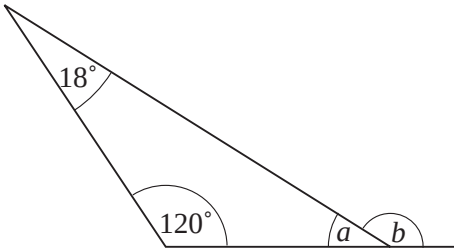
(b)



(c)



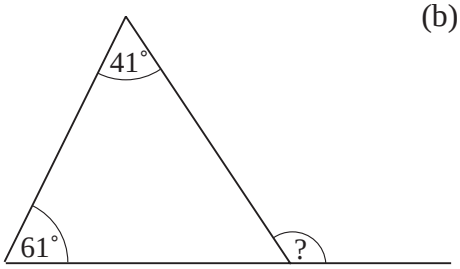
(d)



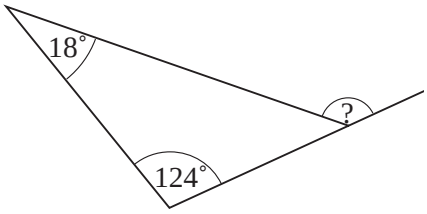
5. Explain how to find the exterior angle without having to calculate an interior angle.

Find the exterior angles marked on these triangles.

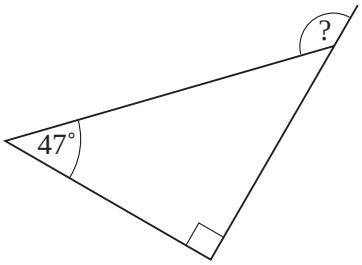
(a)



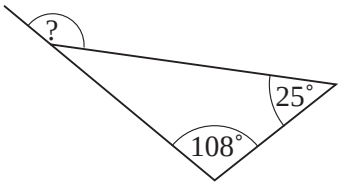
(b)



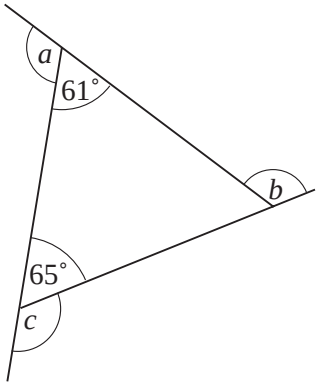
(c)



(d)



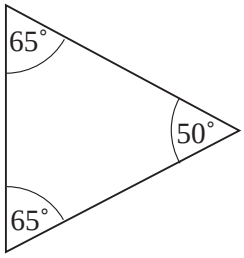
6. Find the total of the 3 exterior angles for this triangle.



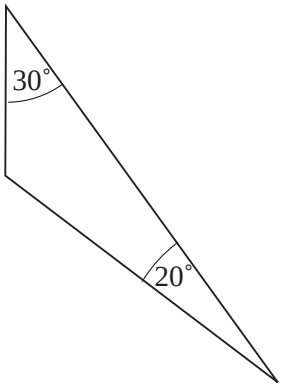
Do you think you will get the same answer for different triangles? Explain your answer.

7. For each of the following triangles, draw in the exterior angles and find their total.

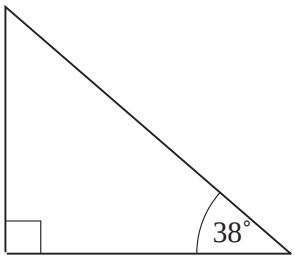
(a)



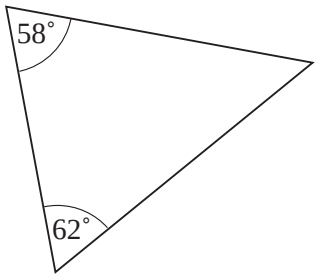
(b)



(c)



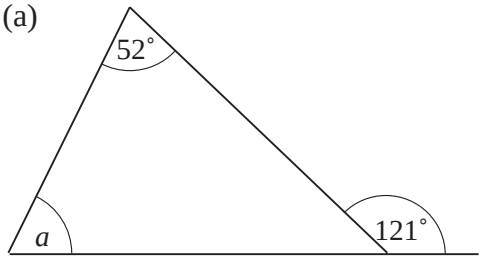
(d)



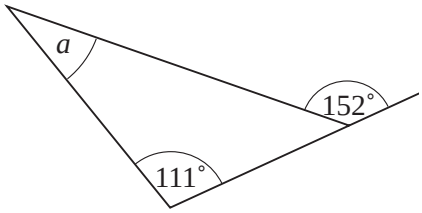
Comment on your results.

8. Find the unknown angle or angles marked in each of the following diagrams.

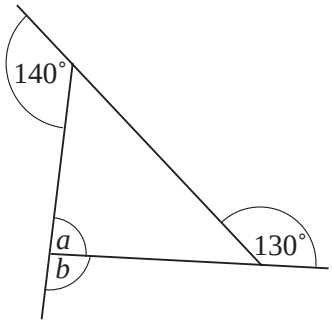
(a)



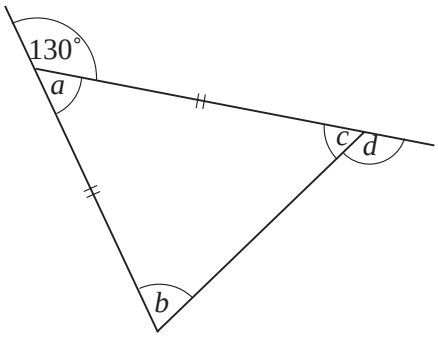
(b)



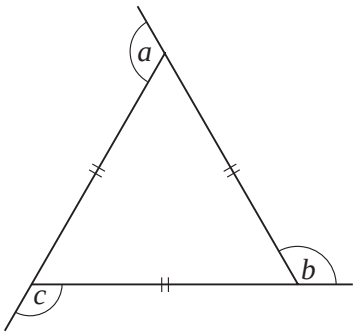
(c)



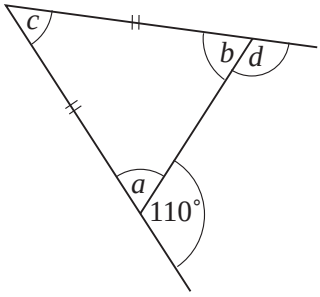
(d)



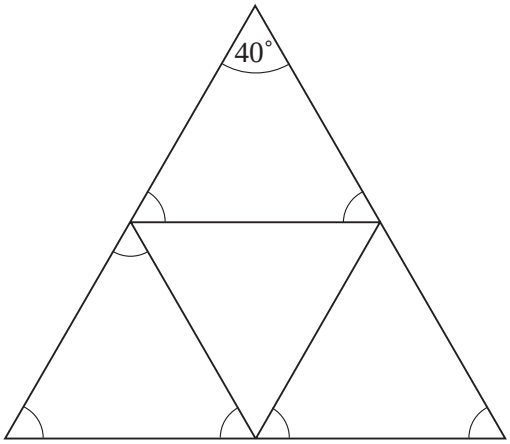
(e)



(f)



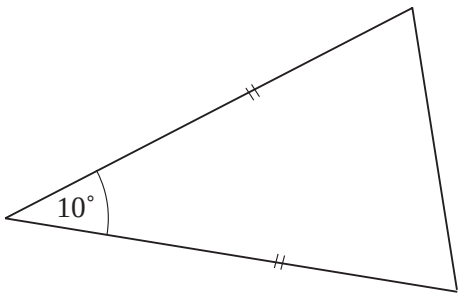
9. Part of a roof is made out of 4 similar isosceles triangles.



Copy the diagram and mark the sides that have the same lengths.

On your diagram, write in the size of all the marked angles.

10. (a) For this isosceles triangle, find the other two interior angles.



(b) Find the other angles if the $10°$ increases to $20°$ and then to $30°$.

(c) What do you think will happen if the $10°$ is increased to $40°$?

11. One angle of an isosceles triangle is $70°$. What are the other angles? (There is more than one solution!)