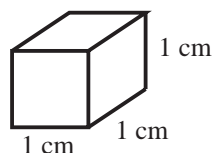


22 Volume

22.1 Concept of Volume: the Unit Cube

In this section we look at volume for the first time, by counting the number of 1 cm cubes in a solid.

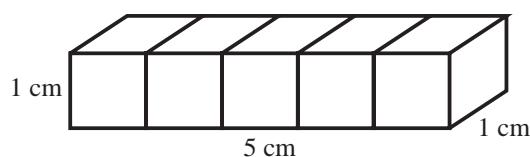


The volume of this cube is 1 cm^3 (1 cubic centimetre)



Example 1

What is the volume of this solid:



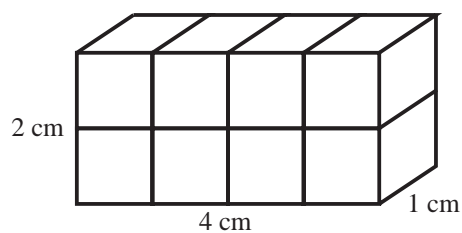
Solution

The solid contains 5 cubes of side 1 cm, so the volume is 5 cm^3 .



Example 2

What is the volume of this solid:



Solution

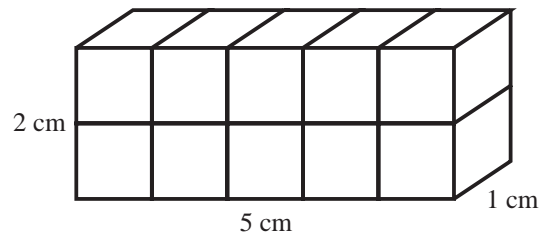
This solid contains 8 cubes of side 1 cm, so the volume is 8 cm^3 .



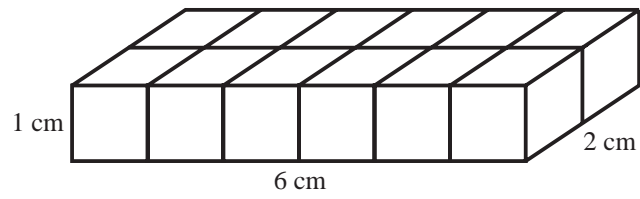
Exercises

1. What is the volume of each of these cuboids:

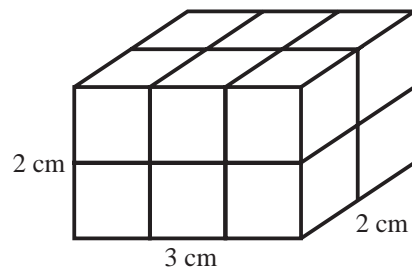
(a)



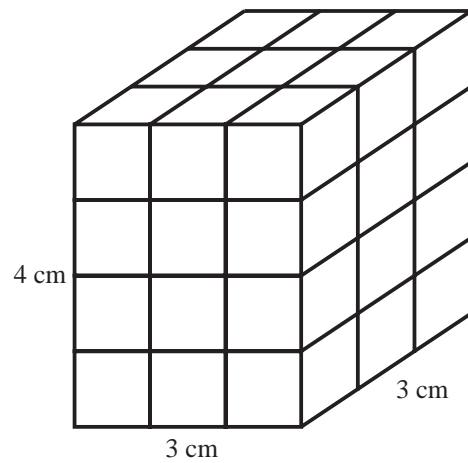
(b)



(c)

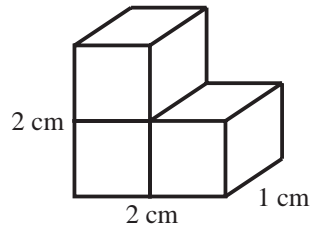


(d)

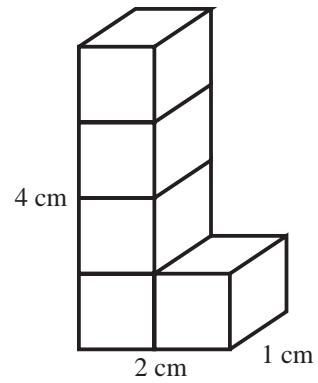


2. What is the volume of each of these solids:

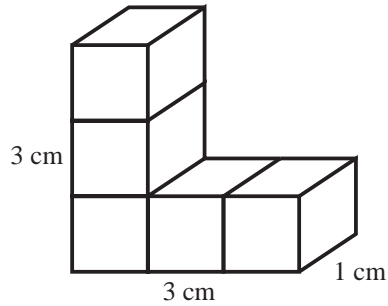
(a)



(b)

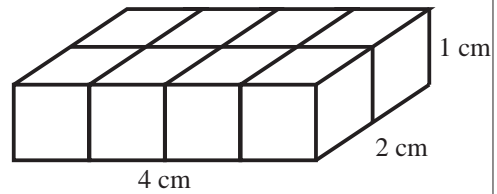


(c)

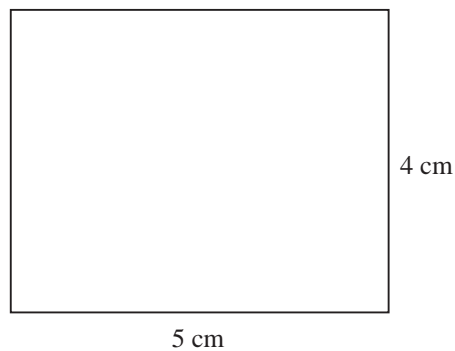


3. The diagram shows the cubes that are used to make the first layer of a cuboid:

- (a) How many cubes are there in the first layer?
- (b) What is the volume of the cuboid if it is made up of 6 layers?

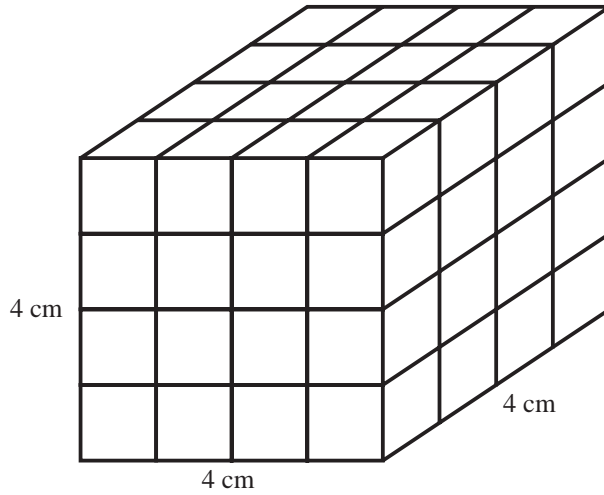


4. A cuboid is built from 1 cm cubes on top of this rectangular base:

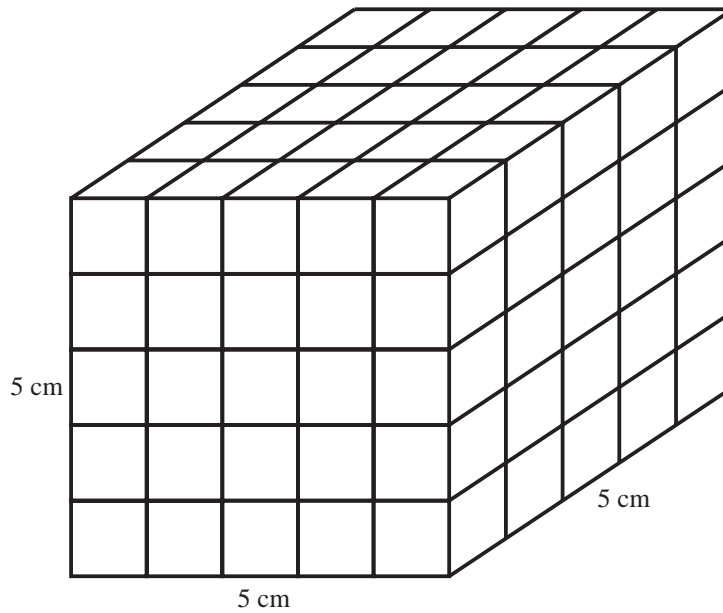


- (a) How many cubes are there in the first layer?
- (b) If there are 4 layers, what is the volume of the cuboid?

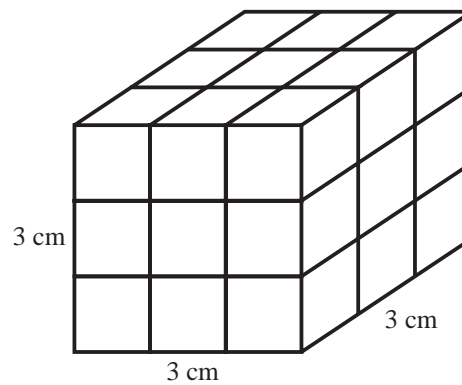
5. The diagram below shows a large cube made from 1 cm cubes.
- (a) How many small cubes are in each layer of the large cube?
 - (b) What is the volume of the large cube?



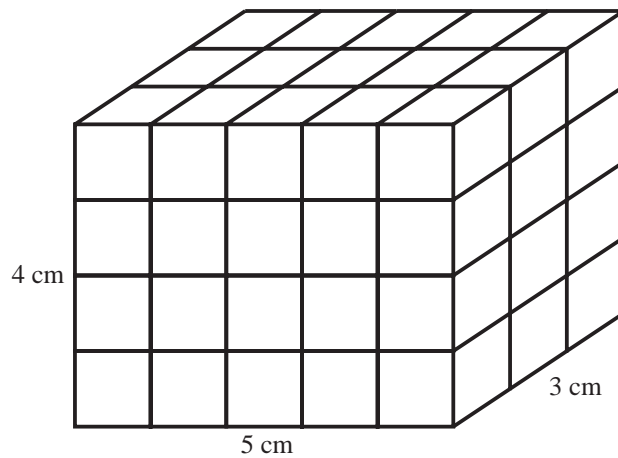
6. What is the volume of this cube:



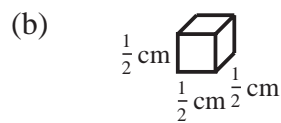
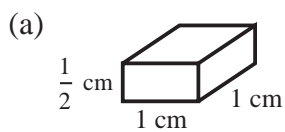
7. (a) What is the volume of the cube shown in the diagram opposite?
- (b) The top layer is cut off. What is the volume of the solid that remains?



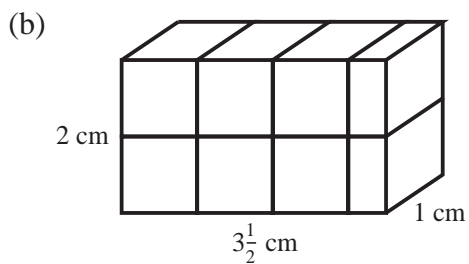
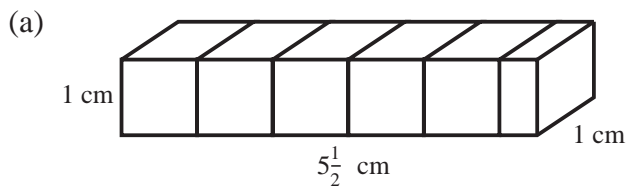
8. Sally is asked to make a cuboid with sides of length 7 cm, 3 cm and 4 cm. She runs out of cubes when she has made the shape in the diagram:



- (a) What is the volume of the shape she has made?
 (b) How many more cubes would she need to make the required shape?
9. How many of each of these shapes would be needed to make a 1 cm cube?
 What is the volume of each of these shapes:



10. What is the volume of each of these shapes:

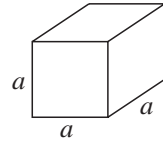


22.2 Volume of a Cube

In this section we consider the volume of a cube and the units of volume.

$$\begin{aligned}\text{Volume of a cube} &= a \times a \times a \\ &= a^3\end{aligned}$$

where a is the length of the each side of the cube



Note: If the sides of the cube are measured in cm, the volume will be measured in cm^3 .



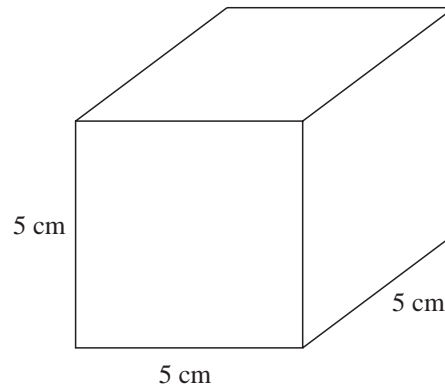
Example 1

What is the volume of this cube:



Solution

$$\begin{aligned}\text{Volume} &= 5^3 \\ &= 5 \times 5 \times 5 \\ &= 125 \text{ cm}^3\end{aligned}$$



Example 2

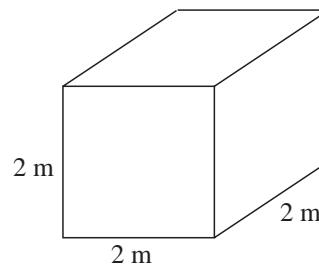
What is the volume of this cube in:

- (a) m^3 ,
- (b) cm^3 ?



Solution

- (a) $\begin{aligned}\text{Volume} &= 2^3 \\ &= 2 \times 2 \times 2 \\ &= 8 \text{ m}^3\end{aligned}$
- (b) Remember that $1 \text{ m} = 100 \text{ cm}$, so $2 \text{ m} = 200 \text{ cm}$.



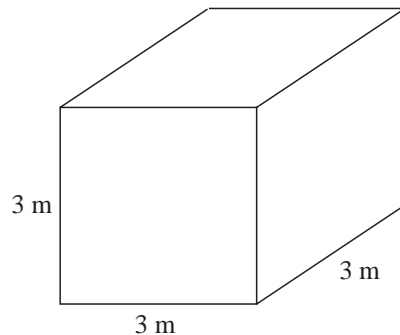
$$1 \text{ m}^3 = 1000000 \text{ cm}^3$$



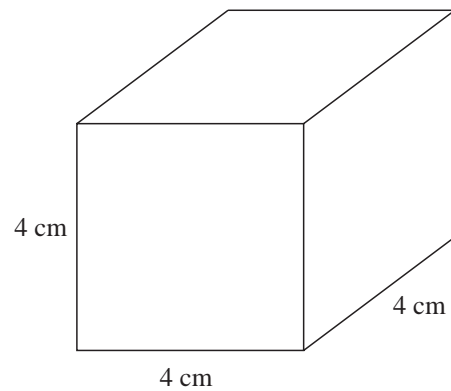
Exercises

1. What is the volume of each of these cubes:

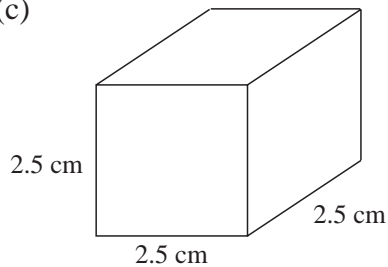
(a)



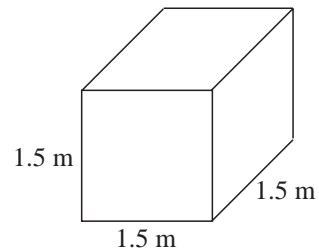
(b)



(c)



(d)



2. A cube has sides of length 30 cm. What is the volume of the cube in:

(a) cm^3 ,

(b) m^3 ?

3. A large box is a cube with sides of length 80 cm. Smaller boxes, which are also cubes, have sides of lengths 20 cm.

(a) What is the volume of the large box?

(b) What is the volume of a small box?

(c) How many small boxes will fit in the large box?

4. A cube has sides of length $\frac{1}{2}$ m.

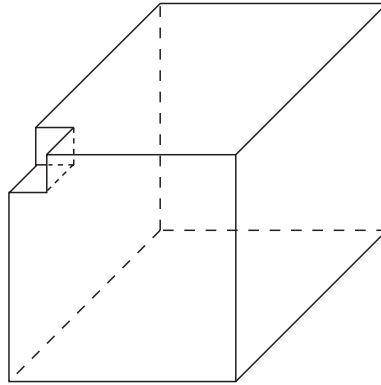
Calculate the volume of the cube:

(a) in m^3 , giving your answer as a fraction,

(b) in m^3 , giving your answer as a decimal,

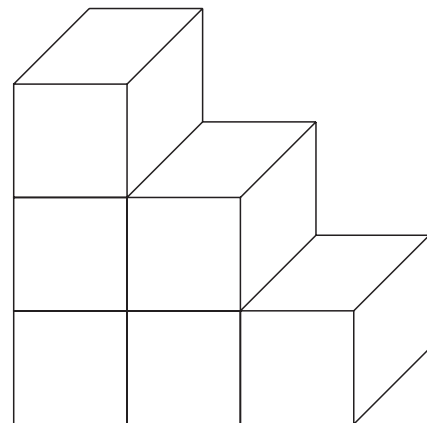
(c) in cm^3 .

5. A cube has sides of length 10 cm. Calculate the volume of the cube in:
- (a) cm^3 , (b) m^3 .
6. The diagram shows a cube with sides of length 30 cm. A smaller cube with sides of length 5 cm has been cut out of the larger cube.



- (a) What is the volume of the large cube before the small cube is cut out?
- (b) What is the volume of the small cube?
- (c) What is the volume of the shape that is left?
7. Wooden building blocks are cubes with sides of length 4 cm. A child builds a tower 6 blocks high. What is the volume of the tower?

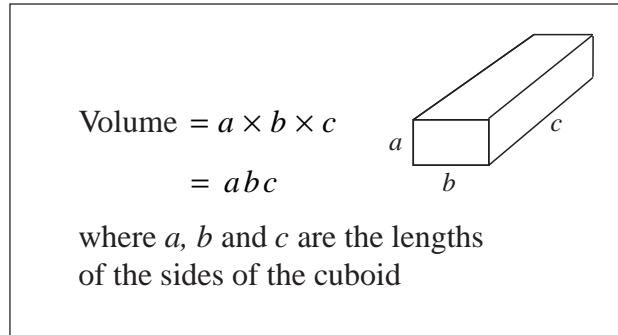
8. This 'staircase' is built from wooden cubes with sides of length 6 cm.
- (a) What is the volume of the staircase?
- (b) A similar staircase is 4 blocks high instead of 3. What is the volume of this staircase?



9. A cube has volume 343 cm^3 .
- (a) How long are the sides of the cube?
- (b) What is the area of one face of the cube?
- (c) What is the total area of the surface of the cube?
10. The area of one face of a cube is 81 cm^2 .
What is the volume of the cube?

22.3 Volume of a Cuboid

We now consider the volume of any cuboid.



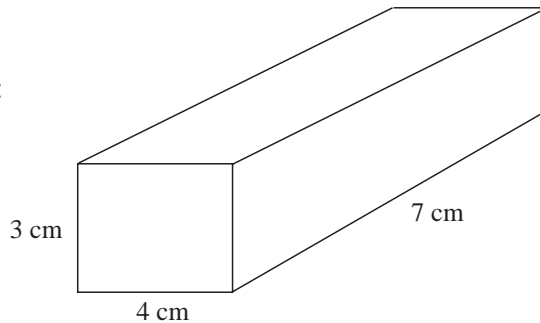
Example 1

Calculate the volume of this cuboid:



Solution

$$\begin{aligned} \text{Volume} &= 3 \times 4 \times 7 \\ &= 84 \text{ cm}^3 \end{aligned}$$



Example 2

A letter 'T' shape is made by sticking together 2 cuboids as shown in the diagram.

What is the total volume of the letter 'T' ?



Solution

First find the volume of the top cuboid:

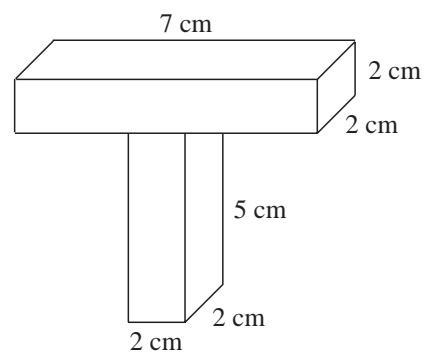
$$\begin{aligned} \text{Volume} &= 7 \times 2 \times 2 \\ &= 28 \text{ cm}^3 \end{aligned}$$

Then find the volume of the upright cuboid:

$$\begin{aligned} \text{Volume} &= 2 \times 2 \times 5 \\ &= 20 \text{ cm}^3 \end{aligned}$$

These two volumes can then be added to give the total volume:

$$\begin{aligned} \text{Total volume} &= 28 + 20 \\ &= 48 \text{ cm}^3 \end{aligned}$$

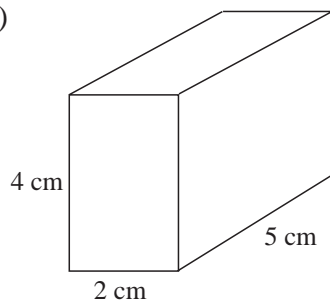




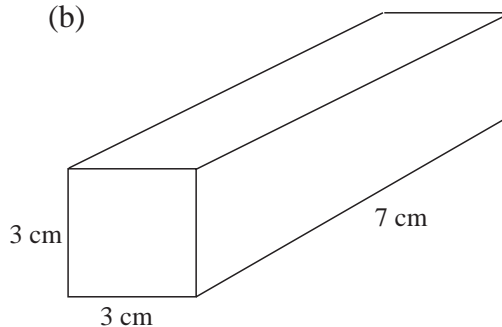
Exercises

1. Calculate the volume of each of these cuboids:

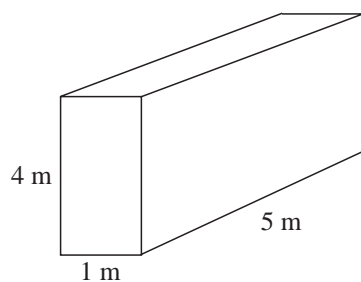
(a)



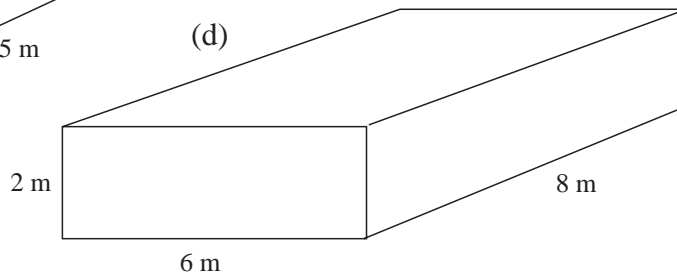
(b)



(c)



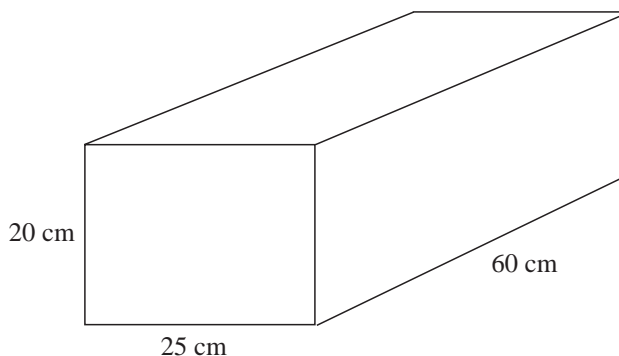
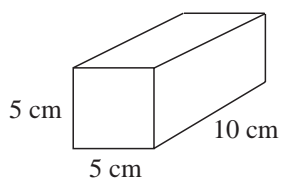
(d)



2. A cuboid has sides of length 5 m, 3 m and 1 m. What is the volume of the cuboid in:

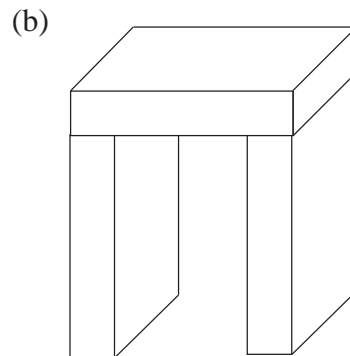
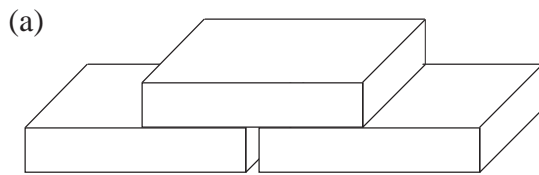
(a) m^3 , (b) cm^3 ?

3. The diagram shows a large box and a small box, both of which are cuboids.



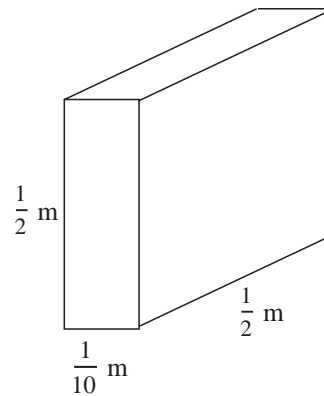
- Calculate the volume of the large box.
- Calculate the volume of the small box.
- How many of the small boxes would fit in the large box?

4. A set of wooden building blocks contains wooden blocks that are 10 cm by 2 cm by 4 cm. The blocks are used to make the shapes below. Calculate the volume of each shape.



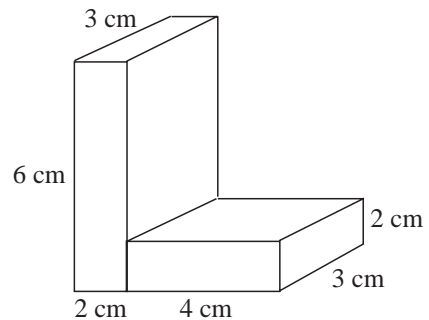
5. Calculate the volume of this cuboid, giving your answer:

- (a) in m^3 , using fractions,
 (b) in m^3 , using decimals,
 (c) in cm^3 .

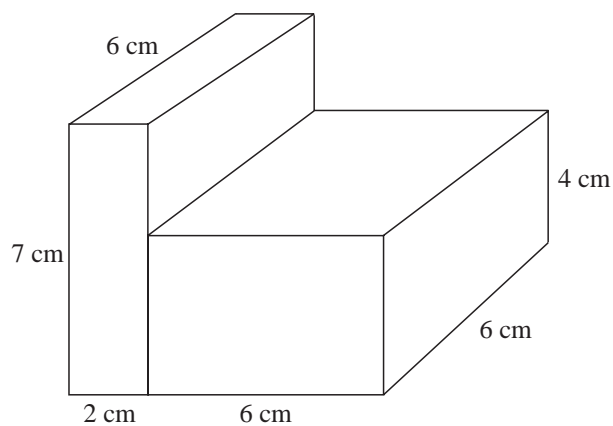


6. A letter 'L' shape is made from two cuboids.

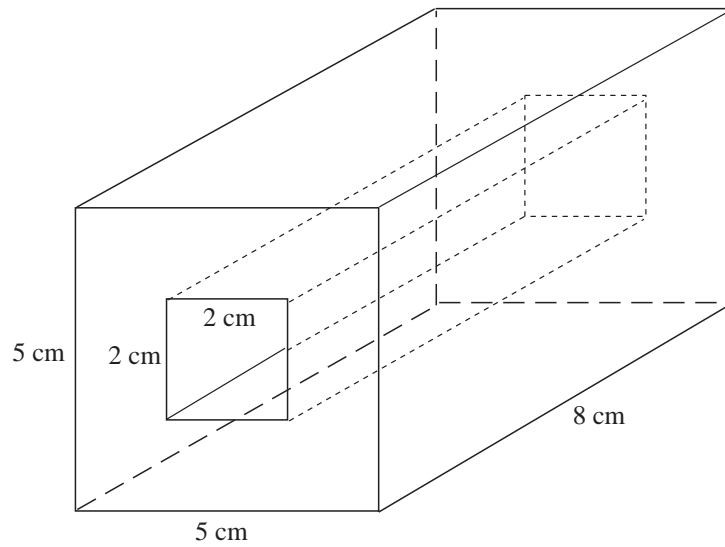
- (a) Calculate the volume of each cuboid.
 (b) Calculate the volume of this letter 'L' shape.



7. Calculate the volume of this solid:

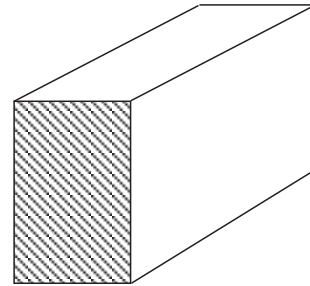


8. The diagram shows a wooden block that has had a square hole cut through it. Calculate the volume of wood in the block.

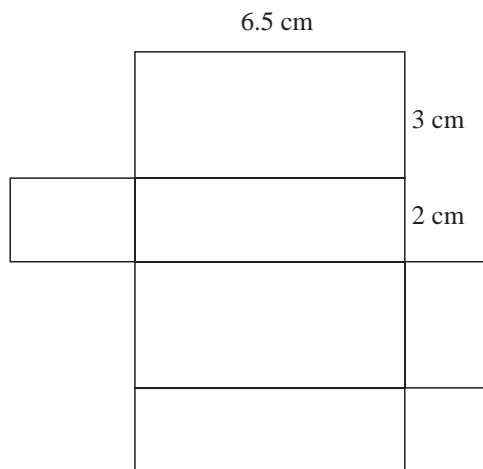


9. The diagram shows a cuboid. The area of the shaded end is 8 cm^2 . How long is the cuboid if its volume is:

- (a) 80 cm^3 ,
- (b) 96 cm^3 ,
- (c) 20 cm^3 ?



10. The shape in the diagram can be folded to form a cuboid. Calculate the volume of the cuboid.



22.4 Capacity

When we refer to how much liquid a tank or container can hold, we often talk about its *capacity* in litres. This is another way of describing its volume.

$$1000 \text{ cm}^3 = 1 \text{ litre}$$



Example 1

What is the capacity, in litres, of a tank with dimensions 1 m by 1 m by 1 m?



Solution

Working in centimetres,

$$\begin{aligned} \text{Volume} &= 100 \times 100 \times 100 \\ &= 1000000 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Capacity (in litres)} &= \frac{1000000}{1000} \\ &= 1000 \text{ litres} \end{aligned}$$

Note that the volume of this tank is also 1 m^3 , so $1 \text{ m}^3 = 1000 \text{ litres}$.

$$1 \text{ m}^3 = 1000 \text{ litres}$$



Example 2

A tank measures 3 m by 4 m by 2 m. What is the capacity of the tank in litres?



Solution

$$\begin{aligned} \text{Volume} &= 3 \times 4 \times 2 \\ &= 24 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} \text{But } 1 \text{ m}^3 &= 1000 \text{ litres} \\ \text{so capacity} &= 24 \times 1000 \\ &= 24\,000 \text{ litres} \end{aligned}$$



Example 3

Calculate the volume of a bottle of capacity 700 cm^3 .



Solution

$$\begin{aligned} \text{Volume} &= \frac{700}{1000} \\ &= 0.7 \text{ litres} \end{aligned}$$



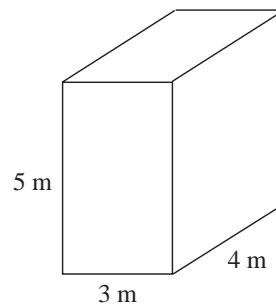
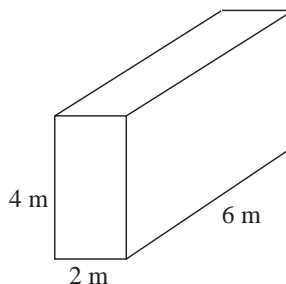
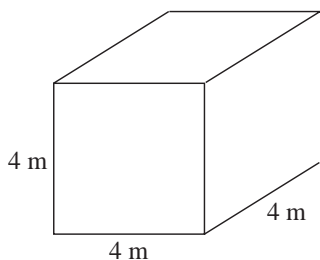
Exercises

- Convert these volumes into cm^3 :

(a) 2 litres	(b) 5 litres	(c) $\frac{1}{2}$ litre
(d) 0.2 litres	(e) 1.5 litres	(f) 2.7 litres
- Convert these volumes into litres:

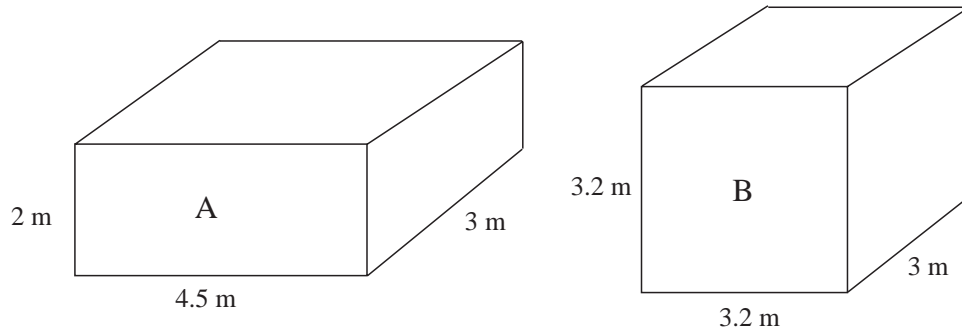
(a) 3000 cm^3	(b) 7000 cm^3	(c) $10\,000 \text{ cm}^3$
(d) 750 cm^3	(e) 250 cm^3	(f) 4900 cm^3
- A tank has dimensions 3 m by 3 m by 2 m.
Calculate the capacity of the tank in:

(a) m^3	(b) litres.
------------------	-------------
- A tank holds 5000 litres. Calculate the volume of the tank in m^3 .
- Work out which of these tanks has the greatest capacity:

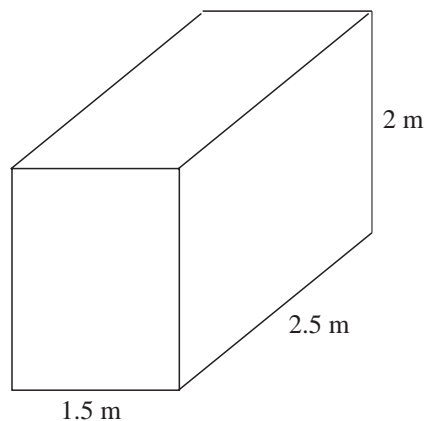


6. A large bottle holds 2 litres of lemonade. The lemonade is poured out into glasses that each hold 25 cm^3 . How many glasses can be filled?

7. The diagram shows 2 different tanks:

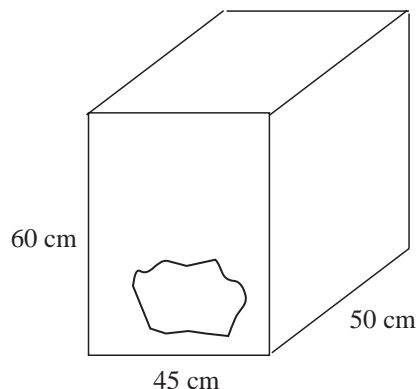


- (a) Which tank has the greater capacity?
- (b) How many more litres does the larger tank hold than the smaller one?
8. The base of a tank is 1.5 m by 2.5 m and its height is 2 m. It is part full of water.



- (a) What is the volume of water in the tank, in litres, if the water is 1.2 m deep?
- (b) How many litres of water does the tank contain when it is $\frac{1}{4}$ full?
- (c) How deep is the water when the tank contains 3000 litres?
9. A tank contains 12.5 litres of liquid. Cans of capacity 800 cm^3 are filled from the tank.
- (a) How many cans can be filled from the tank?
- (b) How much liquid is left over?

10. Ben puts a rock in the tank shown in the diagram and then fills the tank to the top with water. Then he takes the rock out and the water level drops by 5 cm.



- What is the capacity of the tank when it is full?
- What is the volume of water in the tank when the rock has been taken out?
- Calculate the volume of the rock, in cm^3 .
- How many litres of water would be needed to fill the tank to the top again?

22.5 Density

If you were to fill boxes of the same capacity with different materials you would find some easier to lift than others. For example, a box of sand would be much heavier than a box of polystyrene beads. We say that sand is more *dense* than polystyrene. Density, mass and volume are connected by the relationships:

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{Mass} = \text{density} \times \text{volume}$$

$$\text{Volume} = \frac{\text{mass}}{\text{density}}$$

Mercury (the only metal which is liquid at room temperature) has a density of $13\,600 \text{ kg/m}^3$; air has density 1.4 kg/m^3 and water 1000 kg/m^3 or 1 gram/cm^3 .



Example 1

Calculate the mass of 3 litres of water.



Solution

$$3 \text{ litres} = 3000 \text{ cm}^3$$

$$\begin{aligned} \text{Mass} &= \text{density} \times \text{volume} \\ &= 1 \times 3000 \end{aligned}$$

$$= 3000 \text{ grams}$$

$$= 3 \text{ kg}$$

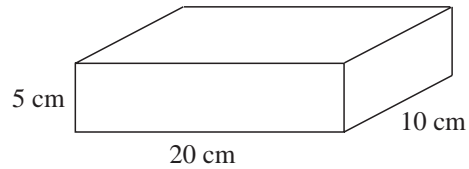


Example 2

This metal block has mass 2 kg.

Calculate the density of the metal in:

- (a) grams/cm³,
- (b) kg/cm³.



Solution

First find the volume of the block:

$$\text{Volume} = 5 \times 20 \times 10$$

$$= 1000 \text{ cm}^3$$

- (a) Note that $2 \text{ kg} = 2000 \text{ grams}$.

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$= \frac{2000}{1000}$$

$$= 2 \text{ grams/cm}^3$$

- (b) Density = $\frac{\text{mass}}{\text{volume}}$
- $$= \frac{2}{1000}$$
- $$= 0.002 \text{ kg/cm}^3$$



Example 3

A type of wood has density 0.7 grams/cm³. A piece of this wood is 3 cm by 10 cm by 180 cm.

What is the mass of this piece of wood, in:

- (a) grams (b) kg?



Solution

- (a) First calculate the volume of the wood:

$$\begin{aligned}\text{Volume} &= 3 \times 10 \times 180 \\ &= 5400 \text{ cm}^3\end{aligned}$$

$$\begin{aligned}\text{Mass} &= \text{density} \times \text{volume} \\ &= 0.7 \times 5400 \\ &= 3780 \text{ grams}\end{aligned}$$

- (b) $3780 \text{ grams} = 3.78 \text{ kg}$

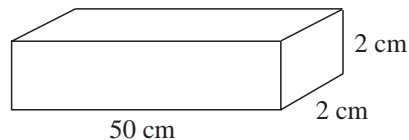


Exercises

1. Calculate the mass of the following volumes of water:

(a) 100 cm^3 (b) 2 litres (c) 0.5 litres.

2. The mass of the metal block below is 3 kg.

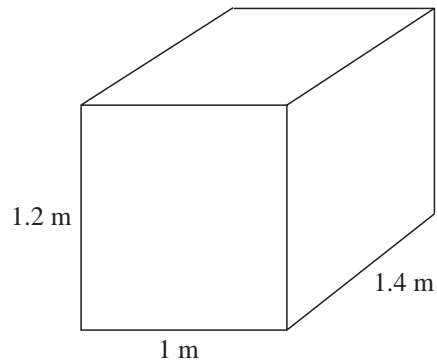


What is the density of the block, in:

- (a) kg/cm^3 ,
 (b) grams/cm^3 ?
3. A polystyrene block has dimensions 1 m by 2 m by 3 m. The mass of the block is 24 kg.
- (a) Calculate the density of the polystyrene in grams/cm^3 .
 (b) A smaller block of polystyrene has dimensions 50 cm by 20 cm by 30 cm. What is its mass?

4. The diagram shows a tank that is to be filled with water. Calculate the mass of water, in kg, if the tank is to be:

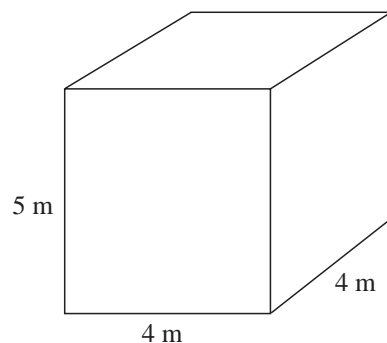
- (a) full,
 (b) $\frac{1}{2}$ full.



5. A rectangular block of metal, 5 cm by 8 cm by 10 cm, has a mass of 500 grams. Calculate the density of the metal in
- (a) g/cm^3 ,
 (b) kg/cm^3 .
6. A book has dimensions 1 cm by 24 cm by 30 cm. Its mass is 576 grams.
- (a) Calculate the density of the book.
 (b) What is the mass of a book with dimensions 1.5 cm by 20 cm by 15 cm?
7. The density of concrete is 4 grams/cm^3 .
- (a) Calculate the mass of a concrete block with dimensions 10 cm by 45 cm by 22 cm.
 (b) Calculate the volume of a concrete block with a mass of 5 kg.
8. A box with dimensions 6 cm by 5 cm by 2 cm is full of soil. The mass of the soil in the box is 72 grams.
- (a) Calculate the density of the soil.
 (b) Calculate the mass of soil, in kg, needed to fill a window box which has dimensions 70 cm by 20 cm by 25 cm.
9. The density of sea water is *not* the same as the density of pure water.

When this tank is filled with sea water the mass of the water is 82 400 kg.

- (a) If the tank was filled with pure water, what would be the mass of the water?
 (b) Does pure water or sea water have the higher density?
 (c) What is the density of sea water in grams/cm^3 ?
 (d) What is the mass of 1 litre of sea water?

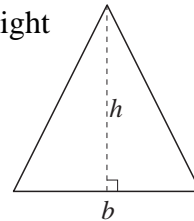


10. One type of metal has a density of 4 grams/cm^3 . Another type has a density of 5 grams/cm^3 . Weights are made from both types of metal. Calculate the difference in volume of 500 gram weights made from the two types of metal.

22.6 Volume of a Triangular Prism

We now look at how to find the volume of a triangular prism. You will need to remember how to find the area of a triangle in order to do this:

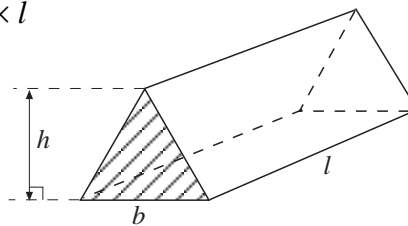
$$\begin{aligned} \text{area of triangle} &= \frac{1}{2} \times \text{base} \times \text{perpendicular height} \\ &= \frac{1}{2} \times b \times h \end{aligned}$$



A triangular prism has the same triangular cross-section throughout its length.

$$\text{Volume of triangular prism} = \text{area of cross-section} \times \text{length}$$

$$= \frac{1}{2} \times b \times h \times l$$



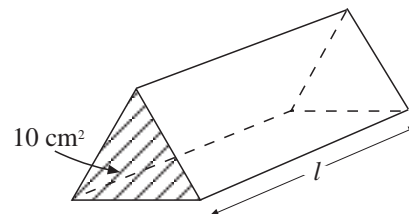
Example 1

The diagram opposite shows a triangular prism.

The area of the end of the prism is 10 cm^2 .

Calculate the volume of the prism, if:

- (a) $l = 5 \text{ cm}$, (b) $l = 2 \text{ m}$.



Solution

- (a) $\text{Volume} = \text{area of cross-section} \times \text{length}$
 $= 10 \times 5$
 $= 50 \text{ cm}^3$

$$(b) \quad 2 \text{ m} = 200 \text{ cm}$$

$$\begin{aligned} \text{Volume} &= \text{area of cross-section} \times \text{length} \\ &= 10 \times 200 \\ &= 2000 \text{ cm}^3 \end{aligned}$$



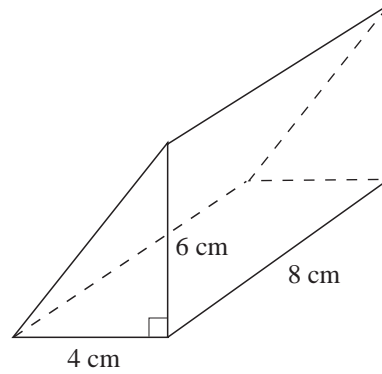
Example 2

Calculate the volume of this triangular prism:



Solution

$$\begin{aligned} \text{Area of cross-section} &= \frac{1}{2} \text{base} \times \text{height} \\ &= \frac{1}{2} \times 4 \times 6 \\ &= 12 \text{ cm}^2 \end{aligned}$$



$$\begin{aligned} \text{Volume of prism} &= \text{area of cross-section} \times \text{length} \\ &= 12 \times 8 \\ &= 96 \text{ cm}^3 \end{aligned}$$



Example 3

The triangular prism opposite has a volume of 82 cm^3 .

Calculate the area of the shaded part of the prism.

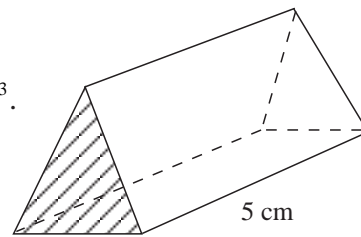


Solution

$$\text{Volume} = \text{area of cross-section} \times \text{length}$$

$$82 = \text{shaded area} \times 5$$

$$\begin{aligned} \text{Shaded area} &= \frac{82}{5} \\ &= 16.4 \text{ cm}^2 \end{aligned}$$

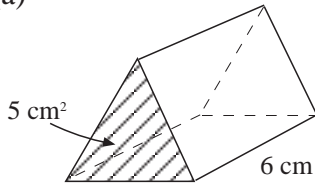




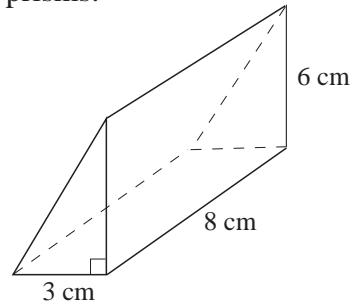
Exercises

1. Calculate the volume of each of these triangular prisms:

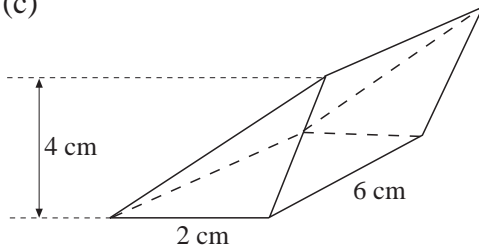
(a)



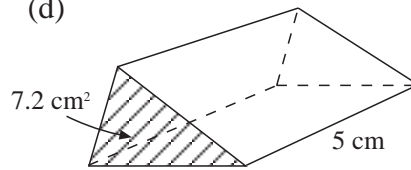
(b)



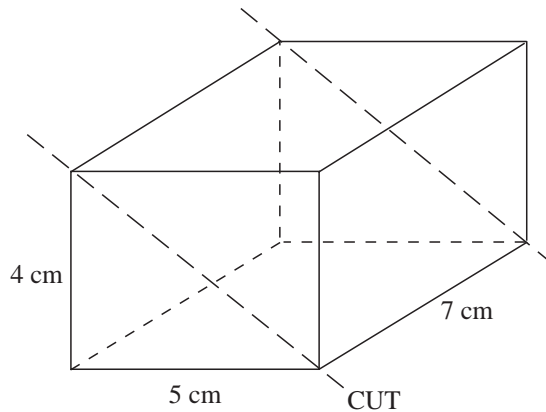
(c)



(d)



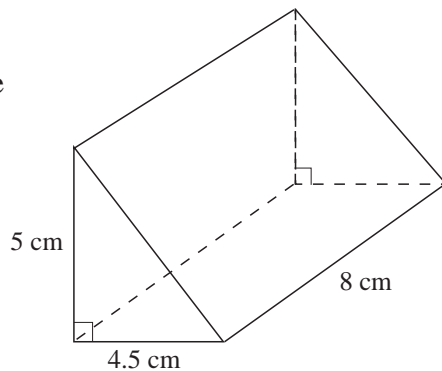
2. Two triangular prisms are formed by cutting the rectangular block below, as shown.



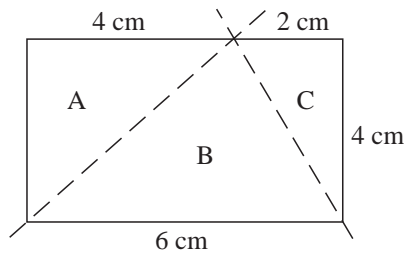
Calculate the volume of each of the triangular prisms formed.

3. Two identical triangular prisms are stuck together to form a rectangular block. One prism is shown opposite.

- (a) What is the volume of this prism?
- (b) What is the volume of the rectangular block formed when the two prisms are stuck together?

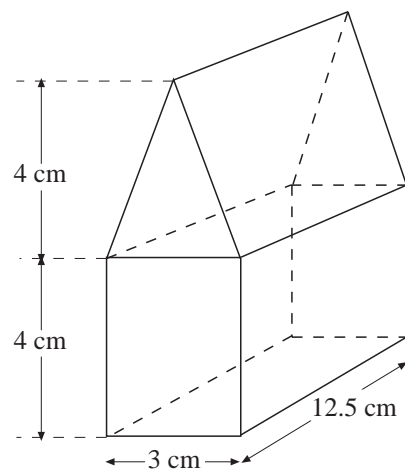


4. A rectangular block has dimension 4 cm by 6 cm by 10 cm. The block is cut to form 3 triangular prisms. The diagram shows how the end of the block is cut.

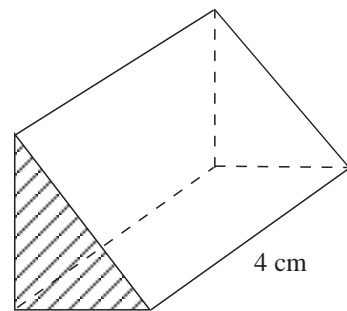


Calculate the volume of each of the triangular prisms.

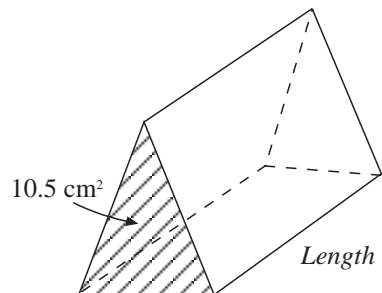
5. A model house is made by sticking a triangular prism on top of a rectangular block, as shown in the diagram.
Calculate the volume of the model house.



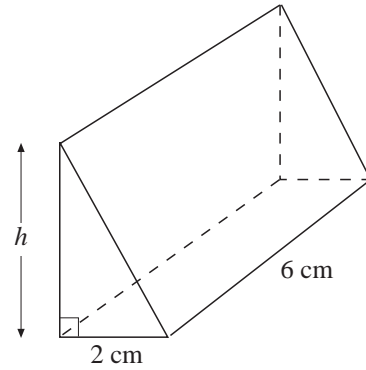
6. The triangular prism opposite has a volume of 84 cm^3 .
- (a) What is the area of the shaded end of the prism?
- (b) If the length was increased to 6 cm, what would now be the volume of the prism?



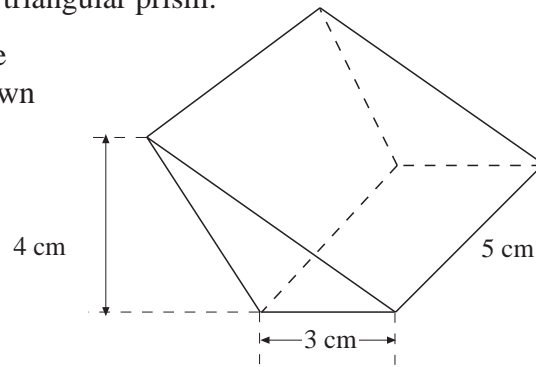
7. The volume of this triangular prism is 63 cm^3 . Find the length of the prism if the area of the shaded end is 10.5 cm^2 .



8. The volume of this triangular prism is 42 cm^3 .
What is the height of the prism?

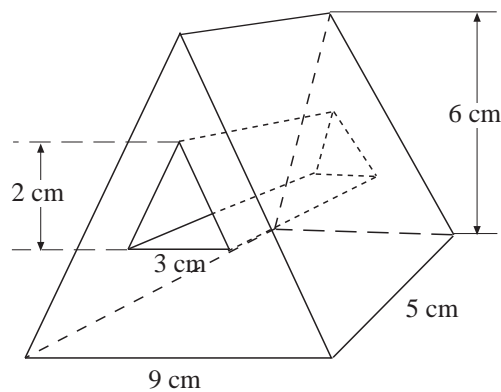


9. (a) Calculate the volume of this triangular prism:
(b) A similar prism has the same shape but all the lengths shown have been *doubled*.



What is the volume of this prism?

- (c) How many times larger is your answer to (b) than your answer to (a)?
(d) Repeat (a), (b) and (c) for any other prism. Do you get the same answer to (c)?
(e) What happens if you *treble* the lengths of the original prism?
(f) What do you think would happen for other enlargements?
10. A triangular prism has a triangular hole cut in it.



Calculate the volume of the prism *after* the hole has been cut out.