

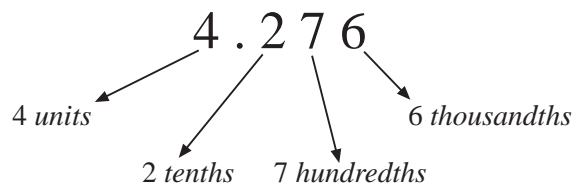
17 Arithmetic: Decimals, Fractions and Percentages

17.1 Conversion: Decimals into Fractions

In this section we revise ideas of decimals and work on writing decimals as fractions.

Recall that the number

means



The table below shows how to write the fractions you need to know in order to write decimals as fractions:

<i>Decimal</i>	<i>Words</i>	<i>Fraction</i>
0.1	1 tenth	$\frac{1}{10}$
0.01	1 hundredth	$\frac{1}{100}$
0.001	1 thousandth	$\frac{1}{1000}$



Example 1

Write these numbers in order, with the smallest first:

0.7, 0.17, 0.77, 0.71, 0.701, 0.107



Solution

Note: It is, perhaps, easier to see this if we first write *all* the numbers to 3 decimal places,

i.e. 0.700, 0.170, 0.770, 0.710, 0.701, 0.107

The required order is:

0.107, 0.17, 0.7, 0.701, 0.71, 0.77



Example 2

Write these numbers as fractions, where possible giving them in their simplest form:

- | | | |
|----------|-----------|-----------|
| (a) 0.7 | (b) 0.09 | (c) 0.004 |
| (d) 0.47 | (e) 0.132 | (f) 1.75 |



Solution

$$(a) \quad 0.7 = \frac{7}{10}$$

$$(b) \quad 0.09 = \frac{9}{100}$$

$$(c) \quad 0.004 = \frac{4}{1000} = \frac{1}{250}$$

$$(d) \quad 0.47 = \frac{47}{100}$$

$$(e) \quad 0.132 = \frac{132}{1000} = \frac{33}{250}$$

$$(f) \quad 1.75 = \frac{175}{100} = \frac{7}{4} \quad (\text{note that fractions larger than 1, such as this, are often referred to as } \textit{improper} \text{ or } \textit{vulgar fractions})$$



Exercises

1. What is the value of the 7 in each of these numbers:

- | | | |
|-----------|-----------|-----------|
| (a) 0.714 | (b) 0.070 | (c) 7.042 |
| (d) 0.007 | (e) 0.471 | (f) 0.157 |

2. Write each list of numbers in order with the smallest first:

- | |
|--|
| (a) 0.61, 0.16, 0.601, 0.106, 0.661, 0.616 |
| (b) 0.47, 0.82, 0.4, 0.78, 0.28 |
| (c) 0.32, 0.23, 0.2, 0.301, 0.3 |
| (d) 0.17, 0.19, 0.9, 0.91, 0.79 |

3. Write each of these decimals as a fraction, giving them in their simplest form:

(a) 0.1	(b) 0.9	(c) 0.3
(d) 0.07	(e) 0.25	(f) 0.001
(g) 0.05	(h) 0.003	(i) 0.017
(j) 0.71	(k) 0.87	(l) 0.201

4. Write each of these decimals as a fraction and simplify where possible:

(a) 0.4	(b) 0.08	(c) 0.54
(d) 0.006	(e) 0.012	(f) 0.162
(g) 0.048	(h) 0.84	(i) 0.328
(j) 0.014	(k) 0.006	(l) 0.108

5. Write down the missing numbers:

(a) $0.6 = \frac{?}{5}$	(b) $0.14 = \frac{?}{50}$	(c) $0.18 = \frac{?}{50}$
(d) $0.008 = \frac{?}{125}$	(e) $0.012 = \frac{?}{250}$	(f) $0.016 = \frac{?}{125}$

6. Write these numbers as *improper fractions* in their simplest form:

(a) 1.2	(b) 3.02	(c) 4.12
(d) 3.62	(e) 4.008	(f) 5.015

7. Calculate, giving your answers as decimals *and* as fractions:

(a) $0.7 + 0.6$	(b) $0.8 - 0.3$
(c) $0.71 + 0.62$	(d) $8.21 - 0.31$
(e) $0.06 + 0.3$	(f) $1.7 + 0.21$
(g) $8.06 - 0.2$	(h) $0.42 - 0.002$

8. Write the missing numbers as decimals and convert them to fractions in their simplest form:

(a) $0.20 + ? = 0.81$	(b) $0.42 + ? = 0.53$
(c) $0.91 - ? = 0.47$	(d) $0.92 - ? = 0.58$

9. Convert these decimals to fractions:

(a) 0.0001

(b) 0.0009

(c) 0.00021

(d) 0.123491

10. Convert these decimals to fractions in their simplest form:

(a) 0.00008

(b) 0.02222

(c) 0.00102

(d) 0.000004

(e) 0.000224

(f) 0.0000002

17.2 Conversion: Fractions into Decimals

In this section we consider how to write fractions as decimals.



Example 1

Write these fractions as decimals:

(a) $\frac{7}{10}$

(b) $\frac{81}{100}$

(c) $\frac{9}{1000}$

(d) $\frac{407}{1000}$



Solution

(a) $\frac{7}{10} = 0.7$

(b) $\frac{81}{100} = 0.81$

(c) $\frac{9}{1000} = 0.009$

(d) $\frac{407}{1000} = 0.407$



Example 2

Write these fractions as decimals:

(a) $\frac{2}{5}$

(b) $\frac{3}{50}$

(c) $\frac{6}{25}$

(d) $\frac{5}{4}$

(e) $\frac{7}{250}$



Solution

In each case, determine the equivalent fraction with the denominator as either 10, 100 or 1000. The fractions can then be written as decimals.

$$(a) \quad \frac{2}{5} = \frac{4}{10} = 0.4 \qquad (b) \quad \frac{3}{50} = \frac{6}{100} = 0.06$$

$$(c) \quad \frac{6}{25} = \frac{24}{100} = 0.24 \qquad (d) \quad \frac{5}{4} = \frac{125}{100} = 1.25$$

$$(e) \quad \frac{7}{250} = \frac{28}{1000} = 0.028$$



Example 3

(a) Calculate $18 \div 5$, then write $\frac{18}{5}$ as a decimal.

(b) Calculate $5 \div 8$, then write $\frac{5}{8}$ as a decimal.



Solution

$$(a) \quad 18 \div 5 = 3.6, \quad \text{since} \quad 5 \overline{)18.0} \begin{array}{r} 3.6 \\ \underline{15} \\ 30 \\ \underline{30} \\ 0 \end{array}$$

$$\begin{aligned} \text{So } \frac{18}{5} &= 18 \div 5 \\ &= 3.6 \end{aligned}$$

$$(b) \quad 5 \div 8 = 0.625, \quad \text{since} \quad 8 \overline{)5.000} \begin{array}{r} 0.625 \\ \underline{4} \\ 10 \\ \underline{8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

$$\begin{aligned} \text{So } \frac{5}{8} &= 5 \div 8 \\ &= 0.625 \end{aligned}$$



Exercises

1. Write these fractions as decimals:

(a) $\frac{3}{10}$

(b) $\frac{7}{100}$

(c) $\frac{9}{1000}$

(d) $\frac{13}{100}$

(e) $\frac{131}{1000}$

(f) $\frac{47}{1000}$

(g) $\frac{21}{100}$

(h) $\frac{183}{1000}$

(i) $\frac{19}{100}$

(j) $\frac{19}{1000}$

(k) $\frac{11}{100}$

(l) $\frac{81}{1000}$

2. Calculate the missing numbers:

(a) $\frac{?}{2} = \frac{5}{10}$

(b) $\frac{?}{20} = \frac{35}{100}$

(c) $\frac{?}{25} = \frac{8}{100}$

(d) $\frac{?}{4} = \frac{25}{100}$

(e) $\frac{2}{?} = \frac{4}{100}$

(f) $\frac{6}{?} = \frac{12}{1000}$

(g) $\frac{8}{?} = \frac{32}{100}$

(h) $\frac{7}{?} = \frac{28}{100}$

3. Write these fractions as decimals:

(a) $\frac{1}{2}$

(b) $\frac{4}{5}$

(c) $\frac{9}{50}$

(d) $\frac{3}{25}$

(e) $\frac{3}{20}$

(f) $\frac{3}{500}$

(g) $\frac{1}{250}$

(h) $\frac{7}{20}$

(i) $\frac{61}{200}$

(j) $\frac{18}{25}$

(k) $\frac{9}{125}$

(l) $\frac{1}{4}$

4. Write these improper fractions as decimals:

(a) $\frac{12}{10}$

(b) $\frac{212}{100}$

(c) $\frac{5218}{1000}$

(d) $\frac{2008}{100}$

(e) $\frac{2008}{1000}$

(f) $\frac{418}{10}$

5. Write these improper fractions as decimals:

(a) $\frac{7}{2}$

(b) $\frac{21}{20}$

(c) $\frac{33}{20}$

(d) $\frac{31}{25}$

(e) $\frac{16}{5}$

(f) $\frac{1001}{500}$

6. Write as a fraction and as a decimal:

(a) $3 \div 5$

(b) $3 \div 8$

(c) $25 \div 4$

(d) $16 \div 5$

(e) $26 \div 4$

(f) $30 \div 8$

7. (a) Calculate $7 \div 8$.

(b) Write $\frac{7}{8}$ as a decimal.

8. (a) Calculate $41 \div 5$.

(b) Write $\frac{41}{5}$ as a decimal.

9. Write $\frac{1}{8}$ as a decimal by using division.

10. Write $\frac{13}{16}$ as a decimal.

17.3 Introduction to Percentages

The word 'percentage' means 'per hundred'. In this section we look at how percentages can be used as an alternative to fractions or decimals.

$$100\% = \frac{100}{100} = 1$$

$$50\% = \frac{50}{100} = \frac{1}{2}$$

$$1\% = \frac{1}{100}$$



Example 1

Draw diagrams to show:

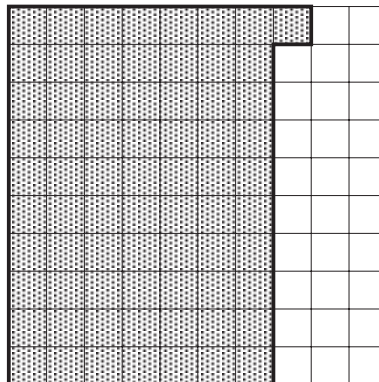
- (a) 71%
- (b) 20%
- (c) 5%



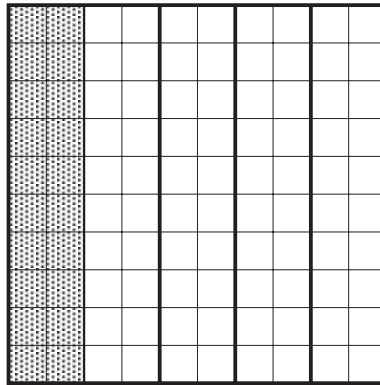
Solution

These percentages can be shown by shading a suitable fraction of a 10 by 10 square shape.

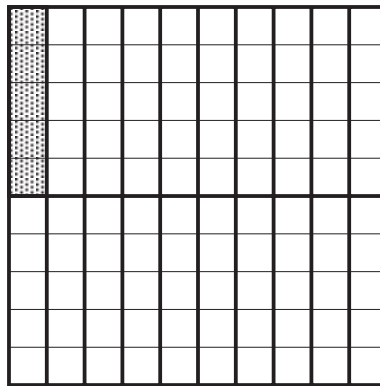
- (a) $71\% = \frac{71}{100}$, so $\frac{71}{100}$ of a shape needs to be shaded:



- (b) $20\% = \frac{20}{100} = \frac{1}{5}$, so $\frac{1}{5}$ of a shape needs to be shaded:



- (c) $5\% = \frac{5}{100} = \frac{1}{20}$, so $\frac{1}{20}$ of a shape needs to be shaded:



Example 2

- (a) What percentage of this shape is shaded?
 (b) What percentage of this shape is *not* shaded?



Solution

- (a) $\frac{3}{5}$ of the shape is shaded, and

$$\begin{aligned}\frac{3}{5} &= \frac{6}{10} \\ &= \frac{60}{100},\end{aligned}$$

so 60% is shaded.

- (b) $100 - 60 = 40$, so 40% is *not* shaded.



Example 3

Find:

- (a) 5% of 100 kg,
- (b) 20% of 40 m,
- (c) 25% of £80.



Solution

$$\begin{aligned}
 \text{(a) } 5\% \text{ of } 100 \text{ kg} &= \frac{5}{100} \times 100 \\
 &= \frac{1}{20} \times 100 \\
 &= 5 \text{ kg}
 \end{aligned}$$

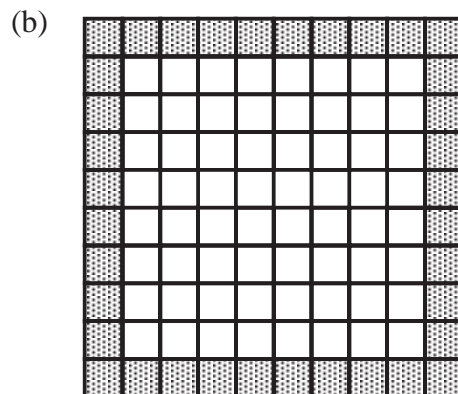
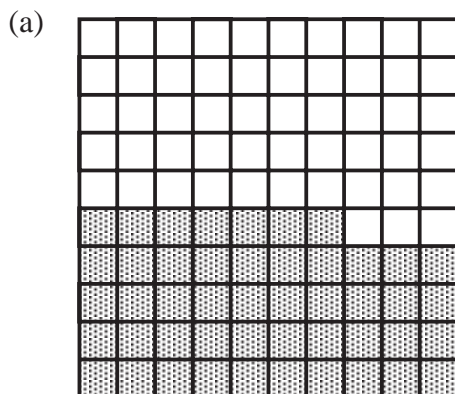
$$\begin{aligned}
 \text{(b) } 20\% \text{ of } 40 \text{ m} &= \frac{20}{100} \times 40 \\
 &= \frac{1}{5} \times 40 \\
 &= 8 \text{ m}
 \end{aligned}$$

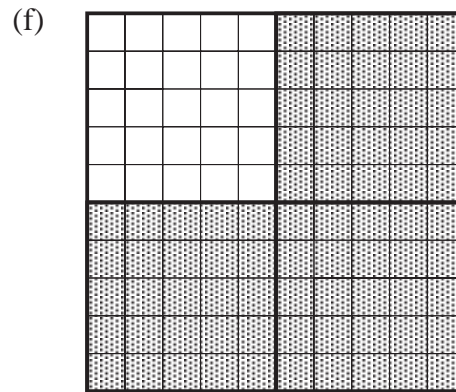
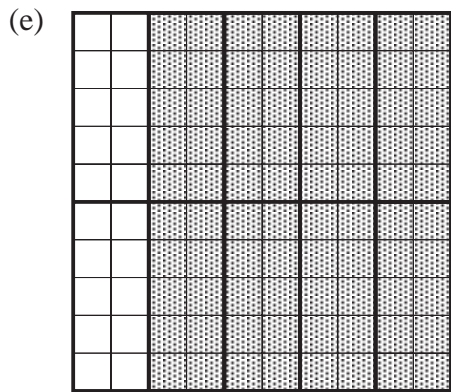
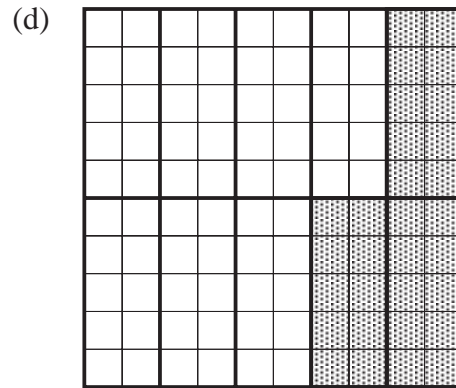
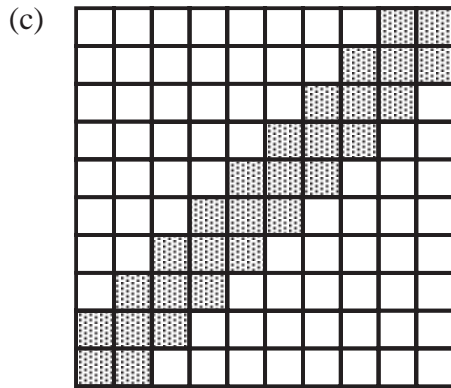
$$\begin{aligned}
 \text{(c) } 25\% \text{ of } £80 &= \frac{25}{100} \times 80 \\
 &= \frac{1}{4} \times 80 \\
 &= £20
 \end{aligned}$$



Exercises

1. For each diagram, state the percentage that is shaded:

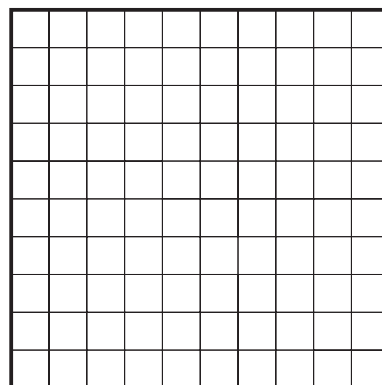




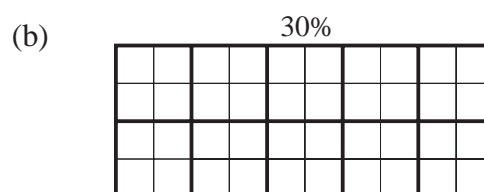
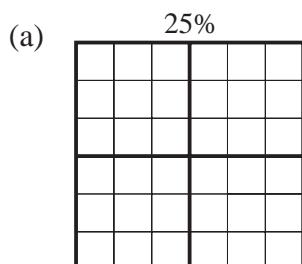
2. For each diagram in question 1, state the percentage that is *not* shaded.
3. If 76% of a rectangle is shaded, what percentage is *not* shaded?

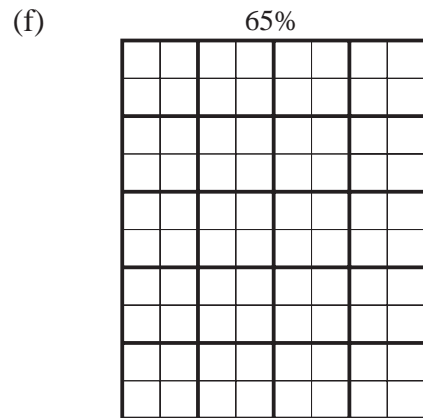
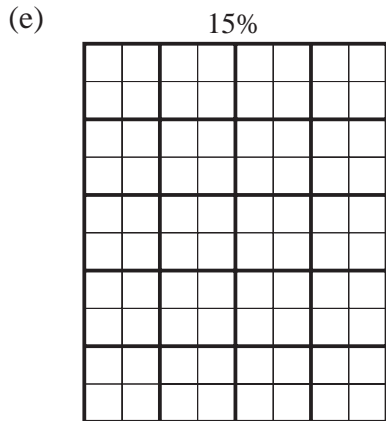
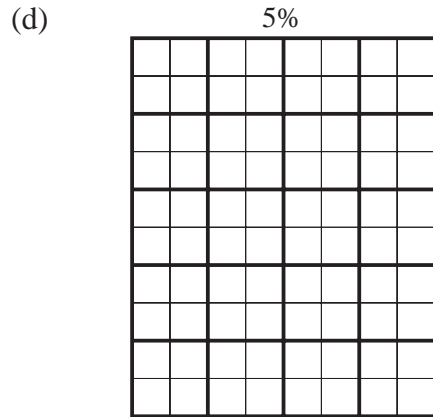
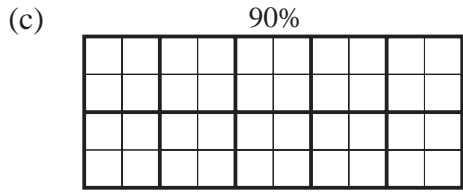
4. Make 4 copies of this diagram and shade the percentage stated:

- (a) 23%
- (b) 50%
- (c) 79%
- (d) 87%

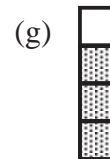
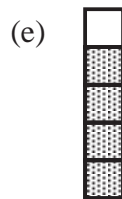
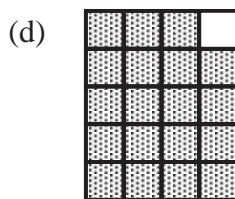
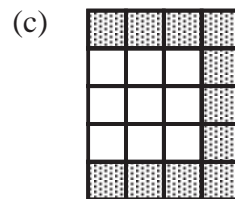
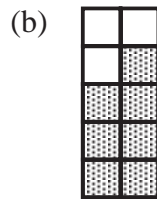
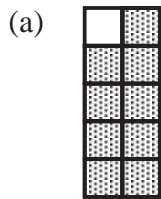


5. Copy each diagram and shade the percentage stated:





6. State the shaded percentage of each shape:



7. If 35% of a class are girls, what percentage are boys?

8. If 88% of a class pass a maths test, what percentage fail the test?

9. Calculate:

(a) 50% of £200,

(b) 30% of 500 kg,

(c) 60% of 50p,

(d) 5% of £2,

- (e) 15% of 10 kg, (f) 25% of 120 m,
 (g) 2% of £400, (h) 26% of £2,
 (i) 20% of £300, (j) 75% of 200 kg.

10. Ben and Adam spend their Saturdays cleaning cars. They agree that Adam will have 60% of the money they earn and that Ben will have the rest.
- (a) What percentage of the money will Ben have?
 (b) How much do they each have if they earn £25?
 (c) How much do they each have if they earn £30?

17.4 Decimals, Fractions and Percentages

In this section we concentrate in converting between decimals, fractions and percentages.



Example 1

Write these percentages as decimals:

- (a) 72% (b) 3%



Solution

$$\begin{aligned} \text{(a)} \quad 72\% &= \frac{72}{100} \\ &= 0.72 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 3\% &= \frac{3}{100} \\ &= 0.03 \end{aligned}$$



Example 2

Write these decimals as percentages:

- (a) 0.71 (b) 0.4 (c) 0.06



Solution

$$\begin{aligned} \text{(a)} \quad 0.71 &= \frac{71}{100} \\ &= 71\% \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 0.4 &= \frac{4}{10} \\ &= \frac{40}{100} \\ &= 40\% \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 0.06 &= \frac{6}{100} \\ &= 6\% \end{aligned}$$



Example 3

Write these percentages as fractions in their simplest possible form:

$$\text{(a)} \quad 90\% \qquad \text{(b)} \quad 20\% \qquad \text{(c)} \quad 5\%$$



Solution

$$\begin{aligned} \text{(a)} \quad 90\% &= \frac{90}{100} \\ &= \frac{9}{10} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 20\% &= \frac{20}{100} \\ &= \frac{1}{5} \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 5\% &= \frac{5}{100} \\ &= \frac{1}{20} \end{aligned}$$



Example 4

Write these fractions as percentages:

$$\text{(a)} \quad \frac{1}{2} \qquad \text{(b)} \quad \frac{2}{5} \qquad \text{(c)} \quad \frac{7}{20}$$



Solution

$$\begin{aligned} \text{(a)} \quad \frac{1}{2} &= \frac{50}{100} \\ &= 50\% \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \frac{2}{5} &= \frac{40}{100} \\ &= 40\% \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \frac{7}{20} &= \frac{35}{100} \\ &= 35\% \end{aligned}$$



Exercises

1. Write these percentages as decimals:

- | | | |
|---------|---------|---------|
| (a) 42% | (b) 37% | (c) 20% |
| (d) 5% | (e) 8% | (f) 10% |
| (g) 22% | (h) 3% | (i) 15% |

2. Write these decimals as percentages:

- | | | |
|----------|----------|----------|
| (a) 0.14 | (b) 0.72 | (c) 0.55 |
| (d) 0.4 | (e) 0.03 | (f) 0.9 |
| (g) 0.18 | (h) 0.04 | (i) 0.7 |

3. Write these percentages as fractions in their simplest forms:

- | | | |
|---------|---------|---------|
| (a) 50% | (b) 30% | (c) 80% |
| (d) 70% | (e) 15% | (f) 25% |
| (g) 64% | (h) 98% | (i) 56% |

4. Write these fractions as percentages:

- | | | |
|---------------------|----------------------|--------------------|
| (a) $\frac{7}{100}$ | (b) $\frac{18}{100}$ | (c) $\frac{3}{50}$ |
| (d) $\frac{17}{50}$ | (e) $\frac{3}{20}$ | (f) $\frac{7}{25}$ |
| (g) $\frac{3}{5}$ | (h) $\frac{7}{10}$ | (i) $\frac{3}{4}$ |
| (j) $\frac{1}{20}$ | (k) $\frac{1}{2}$ | (l) $\frac{3}{25}$ |

5. Copy and complete this table:

<i>Fraction</i>	<i>Decimal</i>	<i>Percentage</i>
	0.04	
		10%
$\frac{1}{2}$		
		45%
$\frac{7}{50}$		
	0.84	

6. There are 200 children in a school hall, eating lunch. Of these children, 124 have chosen chips as part of their lunch.
- What *fraction* of the children have chosen chips?
 - What *percentage* of the children have chosen chips?
 - What percentage of the children have *not* chosen chips?
7. In a survey, $\frac{9}{10}$ of the children in a school said that maths was their favourite subject. What percentage of the children *did not* say that maths was their favourite subject?
8. In a Year 7 class, $\frac{3}{4}$ of the children can swim more than 400 m and only $\frac{1}{10}$ of the children can not swim more than 200 m.
- What percentage of the class can swim:
- more than 400 m,
 - less than 200 m,
 - a distance between 200 m and 400 m?
9. In the school canteen, children can choose chips, baked potato or rice. One day 50% choose chips and 26% choose baked potatoes.
- What percentage choose rice?
 - What fraction of the children choose rice?
10. In a car park, 40% of the cars are red and $\frac{7}{20}$ of the cars are blue.
- What *percentage* are blue?
 - What *percentage* are *neither* red *nor* blue?
 - What *percentage* are red or blue?
 - What *fraction* are red?
 - What *fraction* are *neither* red *nor* blue?
 - What *fraction* are red or blue?